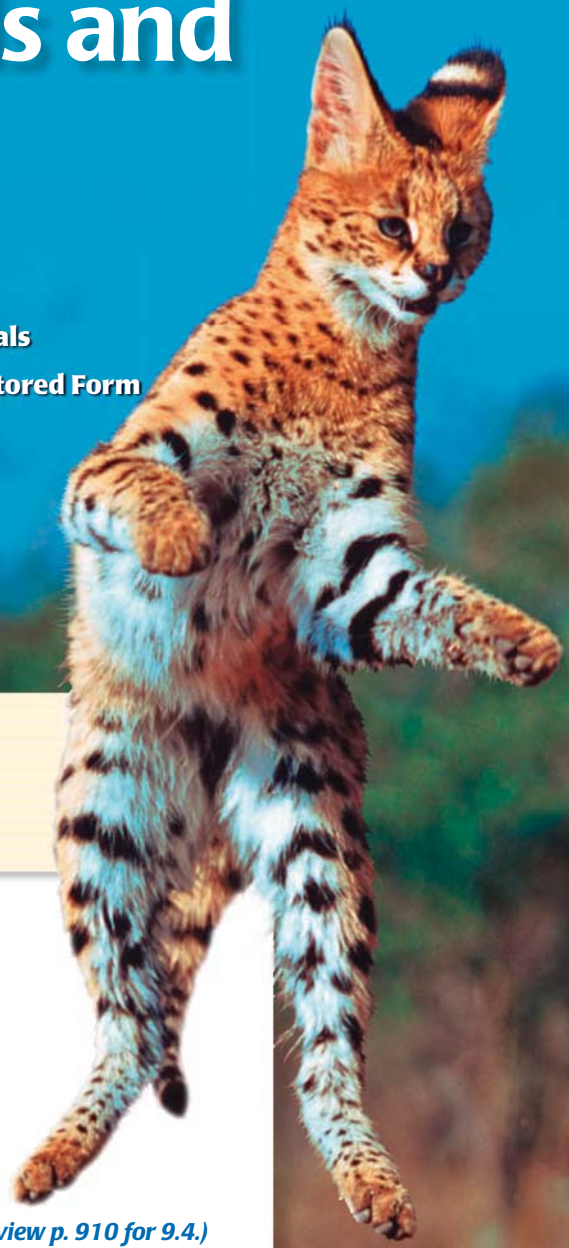
9 Polynomials and Factoring



Before

In previous chapters, you learned the following skills, which you'll use in Chapter 9: using the distributive property, combining like terms, and using the properties of exponents.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. Terms that have the same variable part are called ___?___.
2. For a function $f(x)$, a(n) ___?___ is an x-value for which $f(x) = 0$.

SKILLS CHECK

Find the greatest common factor of the pair of numbers. *(Review p. 910 for 9.4.)*

3. 121, 77 **4.** 96, 32 **5.** 81, 42 **6.** 12, 56

Simplify the expression. *(Review p. 96 for 9.1–9.8.)*

7. $3x + (-6x)$ **8.** $5 + 4x + 2$ **9.** $4(2x - 1) + x$ **10.** $-(x + 4) - 6x$

Simplify the expression. *(Review p. 489 for 9.2–9.8.)*

11. $(3xy)^3$ **12.** $xy^2 \cdot xy^3$ **13.** $(x^5)^3$ **14.** $(-x)^3$

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 9, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 615. You will also use the key vocabulary listed below.

Big Ideas

1. **Adding, subtracting, and multiplying polynomials**
2. **Factoring polynomials**
3. **Writing and solving polynomial equations to solve problems**

KEY VOCABULARY

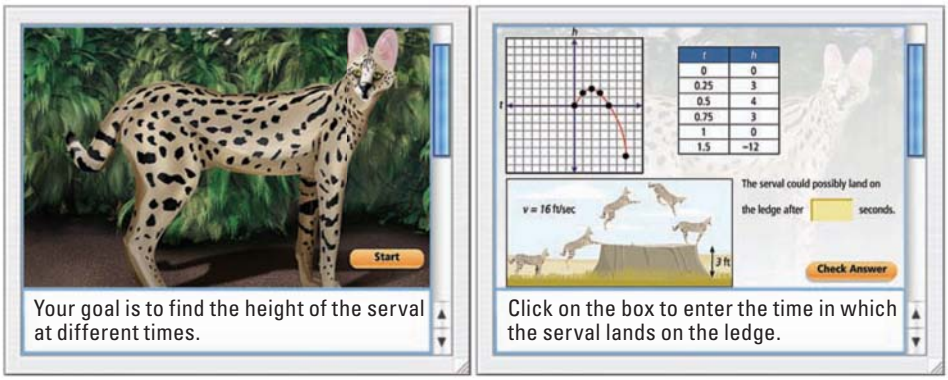
- monomial, *p. 554*
- degree, *p. 554*
- polynomial, *p. 554*
- leading coefficient, *p. 554*
- binomial, *p. 555*
- trinomial, *p. 555*
- roots, *p. 575*
- vertical motion model, *p. 577*
- perfect square trinomial, *p. 601*
- factor by grouping, *p. 606*
- factor completely, *p. 607*

Why?

You can use a polynomial function to model vertical motion. For example, you can use a polynomial function to model the height of a jumping animal as a function of time.

Animated Algebra

The animation illustrated below for Exercise 62 on page 598 helps you to answer this question: How does changing the initial vertical velocity of a serval, an African cat, affect its jumping height?



Algebra at www. publisher.com

Animated Algebra at classzone.com

Other animations for Chapter 9: pages 555, 582, 592, and 601

9.1 Add and Subtract Polynomials

Before You added and subtracted integers.

Now You will add and subtract polynomials.

Why? So you can model trends in recreation, as in Ex. 37.

Key Vocabulary
- **monomial**
- **degree**
- **polynomial**
- **leading coefficient**
- **binomial**
- **trinomial**

A **monomial** is a number, a variable, or the product of a number and one or more variables with whole number exponents. The **degree of a monomial** is the sum of the exponents of the variables in the monomial. The degree of a nonzero constant term is 0. The constant 0 does not have a degree.

Monomial	Degree
10	0
$3x$	1
$\frac{1}{2}ab^2$	$1 + 2 = 3$
$-1.8m^5$	5

Not a monomial	Reason
$5 + x$	A sum is not a monomial.
$\frac{2}{n}$	A monomial cannot have a variable in the denominator.
4^a	A monomial cannot have a variable exponent.
x^{-1}	The variable must have a whole number exponent.

A **polynomial** is a monomial or a sum of monomials, each called a *term* of the polynomial. The **degree of a polynomial** is the greatest degree of its terms.

When a polynomial is written so that the exponents of a variable decrease from left to right, the coefficient of the first term is called the **leading coefficient**.

leading coefficient — degree — constant term

$$2x^3 + x^2 - 5x + 12$$

EXAMPLE 1 Rewrite a polynomial

Write $15x - x^3 + 3$ so that the exponents decrease from left to right. Identify the degree and leading coefficient of the polynomial.

Solution

Consider the degree of each of the polynomial's terms.

Degree is 1. Degree is 3. Degree is 0.

$$15x - x^3 + 3$$

The polynomial can be written as $-x^3 + 15x + 3$. The greatest degree is 3, so the degree of the polynomial is 3, and the leading coefficient is -1.

BINOMIALS AND TRINOMIALS A polynomial with two terms is called a **binomial**. A polynomial with three terms is called a **trinomial**.

EXAMPLE 2 Identify and classify polynomials

Tell whether the expression is a polynomial. If it is a polynomial, find its degree and classify it by the number of its terms. Otherwise, tell why it is not a polynomial.

	Expression	Is it a polynomial?	Classify by degree and number of terms
a.	9	Yes	0 degree monomial
b.	$2x^2 + x - 5$	Yes	2nd degree trinomial
c.	$6n^4 - 8^n$	No; variable exponent	
d.	$n^{-2} - 3$	No; negative exponent	
e.	$7bc^3 + 4b^4c$	Yes	5th degree binomial

ADDING POLYNOMIALS To add polynomials, add like terms. You can use a vertical or a horizontal format.

EXAMPLE 3 Add polynomials

Find the sum.

a. $(2x^3 - 5x^2 + x) + (2x^2 + x^3 - 1)$ **b.** $(3x^2 + x - 6) + (x^2 + 4x + 10)$

Solution

ALIGN TERMS
If a particular power of the variable appears in one polynomial but not the other, leave a space in that column, or write the term with a coefficient of 0.

a. Vertical format: Align like terms in vertical columns.

$$\begin{array}{rr} & 2x^3 - 5x^2 + x \\ + & x^3 + 2x^2 \quad\quad - 1 \\ \hline & 3x^3 - 3x^2 + x - 1 \end{array}$$

b. Horizontal format: Group like terms and simplify.

$$(3x^2 + x - 6) + (x^2 + 4x + 10) = (3x^2 + x^2) + (x + 4x) + (-6 + 10)$$
$$= 4x^2 + 5x + 4$$

Animated Algebra at classzone.com

GUIDED PRACTICE for Examples 1, 2, and 3

1. Write $5y - 2y^2 + 9$ so that the exponents decrease from left to right. Identify the degree and leading coefficient of the polynomial.
2. Tell whether $y^3 - 4y + 3$ is a polynomial. If it is a polynomial, find its degree and classify it by the number of its terms. Otherwise, tell why it is not a polynomial.
3. Find the sum $(5x^3 + 4x - 2x) + (4x^2 + 3x^3 - 6)$.

SUBTRACTING POLYNOMIALS To subtract a polynomial, add its opposite. To find the opposite of a polynomial, multiply each of its terms by -1.

EXAMPLE 4 Subtract polynomials

Find the difference.

a. $(4n^2 + 5) - (-2n^2 + 2n - 4)$ **b.** $(4x^2 - 3x + 5) - (3x^2 - x - 8)$

Solution

a.
$$\begin{array}{r} (4n^2 \quad + 5) \\ -(-2n^2 + 2n - 4) \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 4n^2 \quad + 5 \\ +\ 2n^2 - 2n + 4 \\ \hline 6n^2 - 2n + 9 \end{array}$$

AVOID ERRORS Remember to multiply *each* term in the polynomial by -1 when you write the subtraction as addition.

b. $(4x^2 - 3x + 5) - (3x^2 - x - 8) = 4x^2 - 3x + 5 - 3x^2 + x + 8$

$= (4x^2 - 3x^2) + (-3x + x) + (5 + 8)$

$= x^2 - 2x + 13$

EXAMPLE 5 Solve a multi-step problem

BASEBALL ATTENDANCE Major League Baseball teams are divided into two leagues. During the period 1995–2001, the attendance N and A (in thousands) at National and American League baseball games, respectively, can be modeled by

$N = -488t^2 + 5430t + 24{,}700$ and

$A = -318t^2 + 3040t + 25{,}600$

where t is the number of years since 1995. About how many people attended Major League Baseball games in 2001?

Solution

STEP 1 **Add** the models for the attendance in each league to find a model for M, the total attendance (in thousands).

$M = (-488t^2 + 5430t + 24{,}700) + (-318t^2 + 3040t + 25{,}600)$

$= (-488t^2 - 318t^2) + (5430t + 3040t) + (24{,}700 + 25{,}600)$

$= -806t^2 + 8470t + 50{,}300$

AVOID ERRORS Because a value of M represents *thousands* of people, $M \approx 72{,}100$ represents 72,100,000 people.

STEP 2 **Substitute** 6 for t in the model, because 2001 is 6 years after 1995.

$M = -806(6)^2 + 8470(6) + 50{,}300 \approx 72{,}100$

▶ About 72,100,000 people attended Major League Baseball games in 2001.

GUIDED PRACTICE for Examples 4 and 5

4. Find the difference $(4x^2 - 7x) - (5x^2 + 4x - 9)$.

5. **BASEBALL ATTENDANCE** Look back at Example 5. Find the difference in attendance at National and American League baseball games in 2001.

9.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 21 and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 10, 39, and 41

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A number, a variable, or the product of one or more variables is called a(n) _?_.

2. ★ **WRITING** Is 6 a polynomial? *Explain* why or why not.

EXAMPLE 1 on p. 554 for Exs. 3–9

REWRITING POLYNOMIALS **Write the polynomial so that the exponents decrease from left to right. Identify the degree and leading coefficient of the polynomial.**

3. $9m^5$
4. $2 - 6y$
5. $2x^2y^2 - 8xy$
6. $5n^3 + 2n - 7$
7. $5z + 2z^3 - z^2 + 3z^4$
8. $-2h^2 + 2h^4 - h^6$

9. ★ **MULTIPLE CHOICE** What is the degree of $-4x^3 + 6x^4 - 1$?

   (A) -4 (B) 3 (C) 4 (D) 6

EXAMPLE 2 on p. 555 for Exs. 10–16

10. ★ **MULTIPLE CHOICE** Which expression is *not* a monomial?

    (A) $-5x^2$ (B) $0.2y^4$ (C) $3mn$ (D) $3s^{-2}$

IDENTIFYING AND CLASSIFYING POLYNOMIALS **Tell whether the expression is a polynomial. If it is a polynomial, find its degree and classify it by the number of its terms. Otherwise, tell why it is not a polynomial.**

11. -4^x
12. $w^{-3} + 1$
13. $3x - 5$
14. $\frac{4}{5}f^2 - \frac{1}{2}f + \frac{2}{3}$
15. $6 - n^2 + 5n^3$
16. $10y^4 - 3y^2 + 11$

EXAMPLES 3 and 4 on pp. 555–556 for Exs. 17–28

ADDING AND SUBTRACTING POLYNOMIALS **Find the sum or difference.**

17. $(5a^2 - 3) + (8a^2 - 1)$
18. $(h^2 + 4h - 4) + (5h^2 - 8h + 2)$
19. $(4m^2 - m + 2) + (-3m^2 + 10m + 7)$
20. $(7k^2 + 2k - 6) + (3k^2 - 11k - 8)$
21. $(6c^2 + 3c + 9) - (3c - 5)$
22. $(3x^2 - 8) - (4x^3 + x^2 - 15x + 1)$
23. $(-n^2 + 2n) - (2n^3 - n^2 + n + 12)$
24. $(9b^3 - 13b^2 + b) - (-13b^2 - 5b + 14)$
25. $(4d - 6d^3 + 3d^2) - (9d^3 + 7d - 2)$
26. $(9p^2 - 6p^3 + 3 - 11p) + (7p^3 - 3p^2 + 4)$

ERROR ANALYSIS ***Describe*** **and correct the error in finding the sum or difference of the polynomials.**

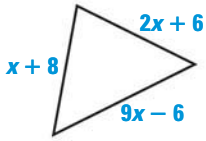
27.
$$\begin{array}{r} x^3 - 4x^2 + 3 \\ +\ -3x^3 + 8x - 2 \\ \hline -2x^3 + 4x^2 + 1 \end{array}$$

28.
$$(6x^2 - 5x) - (2x^2 + 3x - 2)$$
$$= (6x^2 - 2x^2) + (-5x + 3x) - 2$$
$$= 4x^2 - 2x - 2$$

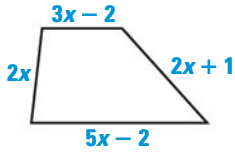
29. **POLYNOMIAL FUNCTIONS** Find the sum $f(x) + g(x)$ and the difference $f(x) - g(x)$ for the functions $f(x) = 3x^2 + x - 7$ and $g(x) = -x^2 + 5x - 2$.

GEOMETRY **Write a polynomial that represents the perimeter of the figure.**

30.



31.



ADDING AND SUBTRACTING POLYNOMIALS **Find the sum or difference.**

32. $(3r^2s + 5rs + 3) + (-8rs^2 - 9rs - 12)$

33. $(x^2 + 11xy - 3y^2) + (-2x^2 - xy + 4y^2)$

34. $(5mn + 3m - 9n) - (13mn + 2m)$

35. $(8a^2b - 6a) - (2a^2b - 4b + 19)$

36. CHALLENGE Consider any integer x. The next consecutive integer can be represented by the binomial $(x + 1)$.

a. Write a polynomial for the sum of any two consecutive integers.

b. *Explain* how you can be sure that the sum of two consecutive integers is always odd. Use the polynomial from part (a) in your explanation.

PROBLEM SOLVING

EXAMPLE 5 on p. 556 for Exs. 37–39

37. BACKPACKING AND CAMPING During the period 1992–2002, the participation B (in millions of people) in backpacking and the participation C (in millions of people) in camping can be modeled by

$$B = -0.0262t^3 + 0.376t^2 - 0.574t + 9.67 \text{ and}$$
$$C = -0.0182t^3 + 0.522t^2 - 2.59t + 47$$

where t is the number of years since 1992. About how many more people camped than backpacked in 2002?

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38. CAR COSTS During the period 1990–2002, the average costs D (in dollars) for a new domestic car and the average costs I (in dollars) for a new imported car can be modeled by

$$D = 442.14t + 14{,}433 \quad \text{and} \quad I = -137.63t^2 + 2705.2t + 15{,}111$$

where t is the number of years since 1990. Find the difference in average costs (in dollars) for a new imported car and a new domestic car in 2002.

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39. ★ SHORT RESPONSE During the period 1998–2002, the number A (in millions) of books for adults and the number J (in millions) of books for juveniles sold can be modeled by

$$A = 9.5t^3 - 58t^2 + 66t + 500 \quad \text{and} \quad J = -15t^2 + 64t + 360$$

where t is the number of years since 1998.

a. Write an equation that gives the total number (in millions) of books for adults and for juveniles sold as a function of the number of years since 1998.

b. Were more books sold in 1998 or in 2002? *Explain* your answer.

○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

40. SCHOOL ENROLLMENT During the period 1985–2012, the projected enrollment B (in thousands of students) in public schools and the projected enrollment R (in thousands of students) in private schools can be modeled by

$$B = -18.53t^2 + 975.8t + 48{,}140 \quad \text{and} \quad R = 80.8t + 8049$$

where t is the number of years since 1985. Write an equation that models the total school enrollment (in thousands of students) as a function of the number of years since 1985. What percent of all students is expected to be enrolled in public schools in 2012?

41. ★ EXTENDED RESPONSE The award for the best pitchers in baseball is named after the pitcher Cy Young. During the period 1890–1911, the total number of Cy Young's wins W and losses L can be modeled by

$$W = -0.44t^2 + 34t + 4.7 \quad \text{and} \quad L = 15t + 15$$

where t is the number of years since 1890.

Cy Young Award

a. A game credited to a pitcher as a win or a loss is called a decision. Write an equation that models the number of decisions for Cy Young as a function of the number of years since 1890.

b. Cy Young's career in Major League Baseball lasted from 1890 to 1911. Approximately how many total decisions did Cy Young have during his career?

c. About what percent of the decisions in Cy Young's career were wins? *Explain* how you found your answer.

42. CHALLENGE In 1970 the United States produced 63.5 quadrillion BTU (British Thermal Units) of energy and consumed 67.86 quadrillion BTU. From 1970 through 2001, the total U.S. energy production increased by about 0.2813 quadrillion BTU per year, and the total U.S. energy consumption increased by about 0.912 quadrillion BTU per year.

a. Write two equations that model the total U.S. energy production and consumption (in quadrillion BTU) as functions of the number of years since 1970.

b. How much more energy was consumed than produced in the U.S. in 1970 and in 2001? What was the change in the amount of energy consumed from 1970 to 2001?

MIXED REVIEW

PREVIEW
Prepare for Lesson 9.2 in Exs. 43–48.

Simplify the expression.

43. $0.6(3 - x)$ *(p. 96)*

44. $4(y + 6)$ *(p. 96)*

45. $4(1 - b) - 5b$ *(p. 96)*

46. $-4(16c - 8)$ *(p. 96)*

47. $(6t^7)^2$ *(p. 489)*

48. $n(2m^2n)$ *(p. 489)*

Graph the equation or inequality.

49. $y = -8$ *(p. 215)*

50. $x - 3y = 15$ *(p. 215)*

51. $y = -5x - 14$ *(p. 215)*

52. $x \geq -3$ *(p. 405)*

53. $x + y \leq 9$ *(p. 405)*

54. $2x - y < 7$ *(p. 405)*

9.1 Graph Polynomial Functions

QUESTION How can you use a graph to check your work with polynomials?

EXAMPLE Check a sum or difference of polynomials

Tell whether the sum or difference is correct.

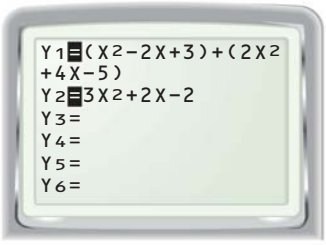
a. $(x^2 - 2x + 3) + (2x^2 + 4x - 5) \stackrel{?}{=} 3x^2 + 2x - 2$

b. $(x^3 + x + 1) - (5x^3 - 2x + 7) \stackrel{?}{=} -4x^3 - x - 6$

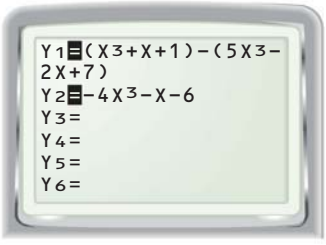
STEP 1 Enter expressions

Let y_1 equal the original expression.
Let y_2 equal the sum.

a.



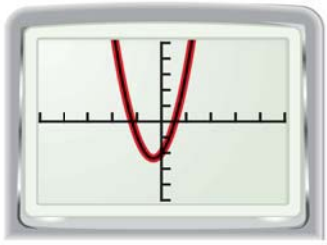
b.



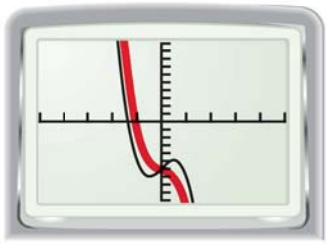
STEP 2 Graph expressions

For y_1, choose a normal graph style.
For y_2, choose a thicker graph style.

a.



b.



STEP 3 Analyze graphs

a. The thick curve coincides with the thin curve, so the sum is correct.

b. The thick curve deviates from the thin curve, so the difference is incorrect.

PRACTICE

Find the sum or difference. Use a graphing calculator to check your answer.

1. $(6x^2 + 4x - 1) + (x^2 - 2x + 2)$ **2.** $(3x^2 - 2x + 1) - (4x^2 - 5x + 1)$

Tell whether the sum or difference is correct. Correct any incorrect answers.

3. $(3x^2 - 2x + 4) + (-x^2 + 3x + 2) \stackrel{?}{=} 2x^2 + x + 6$

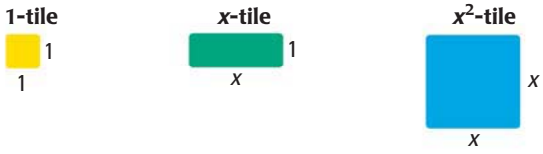
4. $(-4x^2 - 5x - 1) - (-5x^2 + 6x + 3) \stackrel{?}{=} -9x^2 + x + 2$

9.2 Multiplication with Algebra Tiles

MATERIALS • algebra tiles

QUESTION How can you multiply binomials using algebra tiles?

You can use the following algebra tiles to model polynomials. Notice that the value of each tile is the same as its area.

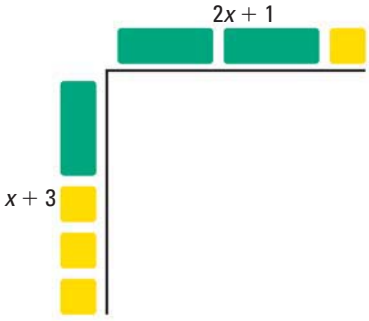


EXPLORE Multiply binomials

Find the product $(x + 3)(2x + 1)$.

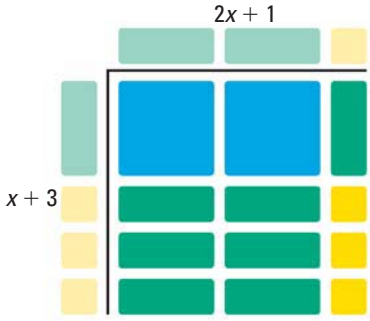
STEP 1 Model the rectangle's dimensions

Model each binomial with algebra tiles. Arrange the first binomial vertically and the second horizontally, as shown. These polynomials model the length and width of a rectangle.



STEP 2 Fill in the area

Fill in the rectangle with the appropriate algebra tiles.



STEP 3 Find the product

The rectangle you created represents the polynomial $2x^2 + 7x + 3$.
So, $(x + 3)(2x + 1) = 2x^2 + 7x + 3$.

DRAW CONCLUSIONS Use your observations to complete these exercises

Use algebra tiles to find the product. Include a drawing of your model.

1. $(x + 1)(x + 3)$ **2.** $(x + 5)(x + 4)$ **3.** $(2x + 1)(x + 2)$

4. $(3x + 2)(x + 1)$ **5.** $(3x + 2)(2x + 1)$ **6.** $(4x + 1)(2x + 3)$

7. REASONING Find the product $x(2x + 1)$ and the product $3(2x + 1)$. What is the sum of these two products? What do your answers suggest you can do to find the product $(x + 3)(2x + 1)$?

9.2 Multiply Polynomials

Before You added and subtracted polynomials.

Now You will multiply polynomials.

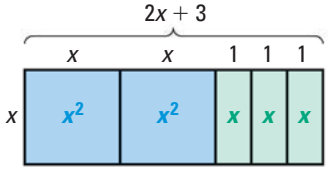
Why? So you can determine areas, as in Example 7.

Key Vocabulary
- **polynomial,** *p. 554*
- **binomial,** *p. 555*

The diagram shows that a rectangle with width x and length $2x + 3$ has an area of $2x^2 + 3x$. You can also find this product by using the distributive property.

$$x(2x + 3) = x(2x) + x(3) = 2x^2 + 3x$$

In this lesson, you will learn several methods for multiplying polynomials. Each method is based on the distributive property.



EXAMPLE 1 Multiply a monomial and a polynomial

REVIEW PROPERTIES OF EXPONENTS
For help with using the properties of exponents, see p. 489.

Find the product $2x^3(x^3 + 3x^2 - 2x + 5)$.

$2x^3(x^3 + 3x^2 - 2x + 5)$ — **Write product.**

$= 2x^3(x^3) + 2x^3(3x^2) - 2x^3(2x) + 2x^3(5)$ — **Distributive property**

$= 2x^6 + 6x^5 - 4x^4 + 10x^3$ — **Product of powers property**

EXAMPLE 2 Multiply polynomials using a table

Find the product $(x - 4)(3x + 2)$.

Solution

STEP 1 **Write** subtraction as addition in each polynomial.

$(x - 4)(3x + 2) = [x + (-4)](3x + 2)$

STEP 2 **Make** a table of products.

	$3x$	2
x	$3x^2$	
-4		

	$3x$	2
x	$3x^2$	$2x$
-4	$-12x$	-8

▶ The product is $3x^2 + 2x - 12x - 8$, or $3x^2 - 10x - 8$.

GUIDED PRACTICE for Examples 1 and 2

Find the product.

1. $x(7x^2 + 4)$ **2.** $(a + 3)(2a + 1)$ **3.** $(4n - 1)(n + 5)$

EXAMPLE 3 Multiply polynomials vertically

Find the product $(b^2 + 6b - 7)(3b - 4)$.

Solution

AVOID ERRORS
Remember that the terms of $(3b - 4)$ are $3b$ and -4. They are *not* $3b$ and 4.

STEP 1 **Multipy** by -4.

$$\begin{array}{r} b^2 + 6b - 7 \\ \times \quad 3b - 4 \\ \hline -4b^2 - 24b + 28 \end{array}$$

STEP 2 **Multiply** by $3b$.

$$\begin{array}{r} b^2 + 6b - 7 \\ \times \quad 3b - 4 \\ \hline -4b^2 - 24b + 28 \\ 3b^3 + 18b^2 - 21b \end{array}$$

STEP 3 **Add** products.

$$\begin{array}{r} b^2 + 6b - 7 \\ \times \quad 3b - 4 \\ \hline -4b^2 - 24b + 28 \\ 3b^3 + 18b^2 - 21b \\ \hline 3b^3 + 14b^2 - 45b + 28 \end{array}$$

EXAMPLE 4 Multiply polynomials horizontally

Find the product $(2x^2 + 5x - 1)(4x - 3)$.

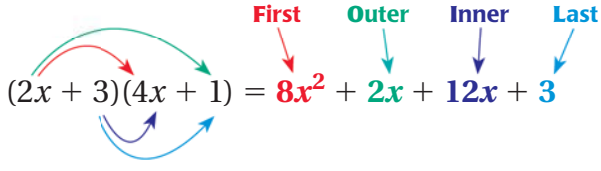
$(2x^2 + 5x - 1)(4x - 3)$ — **Write product.**

$= 2x^2(4x - 3) + 5x(4x - 3) - 1(4x - 3)$ — **Distributive property**

$= 8x^3 - 6x^2 + 20x^2 - 15x - 4x + 3$ — **Distributive property**

$= 8x^3 + 14x^2 - 19x + 3$ — **Combine like terms.**

FOIL PATTERN The letters of the word FOIL can help you to remember how to use the distributive property to multiply binomials. The letters should remind you of the words **F**irst, **O**uter, **I**nner, and **L**ast.



EXAMPLE 5 Multiply binomials using the FOIL pattern

Find the product $(3a + 4)(a - 2)$.

$(3a + 4)(a - 2)$

$= (3a)(a) + (3a)(-2) + (4)(a) + (4)(-2)$ — **Write products of terms.**

$= 3a^2 + (-6a) + 4a + (-8)$ — **Multiply.**

$= 3a^2 - 2a - 8$ — **Combine like terms.**

GUIDED PRACTICE for Examples 3, 4, and 5

Find the product.

4. $(x^2 + 2x + 1)(x + 2)$ **5.** $(3y^2 - y + 5)(2y - 3)$ **6.** $(4b - 5)(b - 2)$

EXAMPLE 6 Standardized Test Practice

The dimensions of a rectangle are $x + 3$ and $x + 2$. Which expression represents the area of the rectangle?

Ⓐ $x^2 + 6$ Ⓑ $x^2 + 5x + 6$ Ⓒ $x^2 + 6x + 6$ Ⓓ $x^2 + 6x$

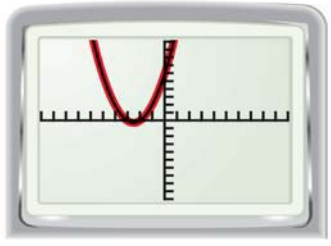
ELIMINATE CHOICES
When you multiply $x + 3$ and $x + 2$, the product will have a constant term of $3 \cdot 2 = 6$. So, you can eliminate choice D.

Solution

Area = length • width	Formula for area of a rectangle
$= (x + 3)(x + 2)$	Substitute for length and width.
$= x^2 + 2x + 3x + 6$	Multiply binomials.
$= x^2 + 5x + 6$	Combine like terms.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

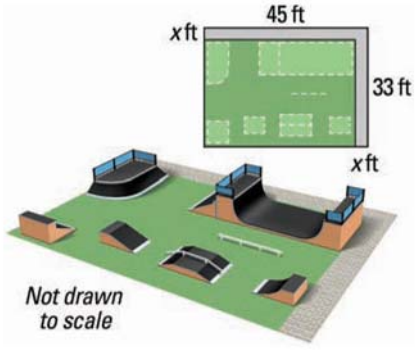
CHECK You can use a graph to check your answer. Use a graphing calculator to display the graphs of $y_1 = (x + 3)(x + 2)$ and $y_2 = x^2 + 5x + 6$ in the same viewing window. Because the graphs coincide, you know that the product of $x + 3$ and $x + 2$ is $x^2 + 5x + 6$.



EXAMPLE 7 Solve a multi-step problem

SKATEBOARDING You are designing a rectangular skateboard park on a lot that is on the corner of a city block. The park will have a walkway along two sides. The dimensions of the lot and the walkway are shown in the diagram.

- Write a polynomial that represents the area of the skateboard park.
- What is the area of the park if the walkway is 3 feet wide?



Solution

STEP 1 **Write** a polynomial using the formula for the area of a rectangle. The length is $45 - x$. The width is $33 - x$.

Area = length • width	Formula for area of a rectangle
$= (45 - x)(33 - x)$	Substitute for length and width.
$= 1485 - 45x - 33x + x^2$	Multiply binomials.
$= 1485 - 78x + x^2$	Combine like terms.

STEP 2 **Substitute** 3 for x and evaluate.

$$\text{Area} = 1485 - 78(3) + (3)^2 = 1260$$

▶ The area of the park is 1260 square feet.

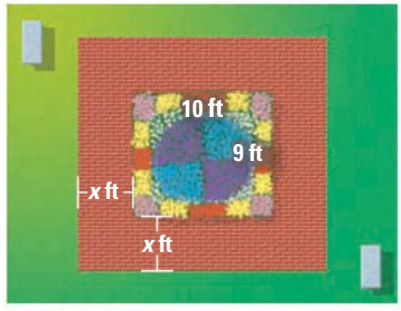
✓ GUIDED PRACTICE for Examples 6 and 7

7. The dimensions of a rectangle are $x + 5$ and $x + 9$. Which expression represents the area of the rectangle?

Ⓐ $x^2 + 45x$ Ⓑ $x^2 + 45$

Ⓒ $x^2 + 14x + 45$ Ⓓ $x^2 + 45x + 45$

8. GARDEN DESIGN You are planning to build a walkway that surrounds a rectangular garden, as shown. The width of the walkway around the garden is the same on every side.



a. Write a polynomial that represents the combined area of the garden and the walkway.

b. Find the combined area when the width of the walkway is 4 feet.

9.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 23 and 51

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 26, 44, 52, and 53

SKILL PRACTICE

1. VOCABULARY Copy and complete: The FOIL pattern can be used to multiply any two ___?___.

2. ★ WRITING *Explain* how the letters of the word FOIL can help you multiply polynomials.

EXAMPLE 1 on p. 562 for Exs. 3–8

MULTIPLYING POLYNOMIALS **Find the product.**

3. $x(2x^2 - 3x + 9)$ **4.** $4y(-y^3 - 2y - 1)$ **5.** $z^2(4z^4 + z^3 - 11z^2 - 6)$

6. $3c^3(8c^4 - c^2 - 3c + 5)$ **7.** $-a^5(-9a^2 + 5a + 13)$ **8.** $-5b^3(4b^5 - 2b^3 + b - 11)$

EXAMPLE 2 on p. 562 for Exs. 9–15

USING TABLES **Use a table to find the product.**

9. $(x + 2)(x - 3)$ **10.** $(y - 5)(2y + 3)$ **11.** $(4b - 3)(b - 7)$

12. $(5s + 2)(s + 8)$ **13.** $(3k - 1)(4k + 9)$ **14.** $(8n - 5)(3n - 6)$

EXAMPLES 3 and 4 on p. 563 for Exs. 16–26

ERROR ANALYSIS ***Describe*** **and correct the error in finding the product of the polynomials.**

15.

$(x - 5)(3x + 1)$

	$3x$	1
x	$3x^2$	x
5	$15x$	5

$(x - 5)(3x + 1) = 3x^2 + 16x + 5$

16.

$$\begin{array}{r} 2x^2 - 3x - 4 \\ \times \qquad x + 7 \\ \hline 14x^2 - 21x - 28 \\ 2x^3 - 3x^2 - 4x \\ \hline 2x^3 + 11x^4 - 25x^2 - 28 \end{array}$$

MULTIPLYING POLYNOMIALS **Use a vertical or a horizontal format to find the product.**

17. $(y + 6)(y - 5)$ **18.** $(5x - 8)(2x - 5)$ **19.** $(7w + 5)(11w - 3)$

20. $(b - 2)(b^2 - b + 1)$ **21.** $(s + 4)(s^2 + 6s - 5)$ **22.** $(-r + 7)(2r^2 - r - 9)$

23. $(5x + 2)(-3x^2 + 4x - 1)$ **24.** $(y^2 + 8y - 6)(4y - 3)$ **25.** $(6z^2 + z - 1)(9z - 5)$

26. ★ **MULTIPLE CHOICE** What is the product of $2x - 9$ and $4x + 1$?

Ⓐ $8x^2 - 38x - 9$ Ⓑ $8x^2 - 34x - 9$

Ⓒ $8x^2 + 34x - 9$ Ⓓ $8x^2 + 38x - 9$

EXAMPLE 5 on p. 563 for Exs. 27–32

USING THE FOIL PATTERN **Use the FOIL pattern to find the product.**

27. $(2r - 1)(5r + 3)$ **28.** $(7a - 2)(3a - 4)$ **29.** $(4m + 9)(2m + 7)$

30. $(8t + 11)(6t - 1)$ **31.** $(4x - 5)(12x - 7)$ **32.** $(8z + 3)(5z + 4)$

SIMPLIFYING EXPRESSIONS **Simplify the expression.**

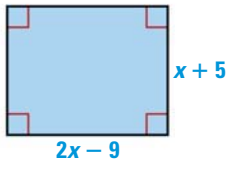
33. $p(2p - 3) + (p - 3)(p + 3)$ **34.** $x^2(7x + 5) - (2x + 6)(x - 1)$

35. $-3c^2(c + 11) - (4c - 5)(3c - 2)$ **36.** $2w^3(2w^3 - 7w - 1) + w(5w^2 + 2w)$

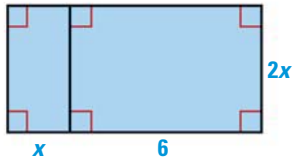
EXAMPLES 6 and 7 on p. 564 for Exs. 37–42

GEOMETRY **Write a polynomial that represents the area of the shaded region.**

37.



38.



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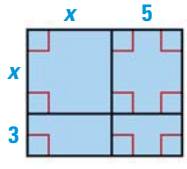

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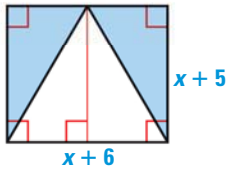
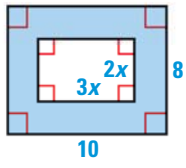
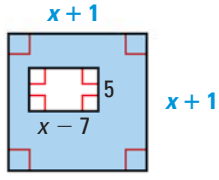


41.



42.



43. **POLYNOMIAL FUNCTIONS** Find the product $f(x) \cdot g(x)$ for the functions $f(x) = x - 11$ and $g(x) = 2x + 12$.

44. ★ **MULTIPLE CHOICE** Which polynomial represents $f(x) \cdot g(x)$ if $f(x) = -2x^2$ and $g(x) = x^3 - 5x^2 + 2x - 1$?

Ⓐ $-2x^5 - 10x^4 + 4x^3 - 2x^2$ Ⓑ $-2x^5 + 10x^4 - 4x^3 - 2x^2$

Ⓒ $-2x^5 + 10x^4 - 4x^3 + 2x^2$ Ⓓ $2x^5 - 10x^4 + 4x^3 - 2x^2$

45. **REASONING** Find the product $(x^2 - 7x)(2x^2 + 3x + 1)$. Show that the product is correct by using a graphing calculator. *Explain* your reasoning.

CHALLENGE **Find the product.**

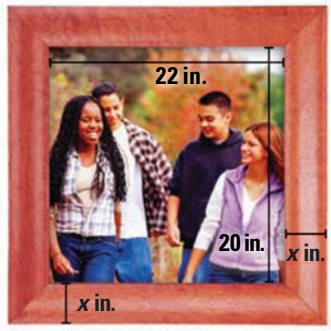
46. $(x - y)(3x + 4y)$ **47.** $(x^2y + 9y)(2x + 3y)$ **48.** $(x^2 - 5xy + y^2)(4xy)$

◯ = WORKED-OUT SOLUTIONS p. WS1 ★ = STANDARDIZED TEST PRACTICE

PROBLEM SOLVING

EXAMPLE 7 on p. 564 for Exs. 49–50

49. PICTURE FRAME You are designing a frame to surround a rectangular picture. The width of the frame around the picture is the same on every side, as shown.



a. Write a polynomial that represents the total area of the picture and the frame.

b. Find the combined area of the picture and the frame when the width of the frame is 4 inches.

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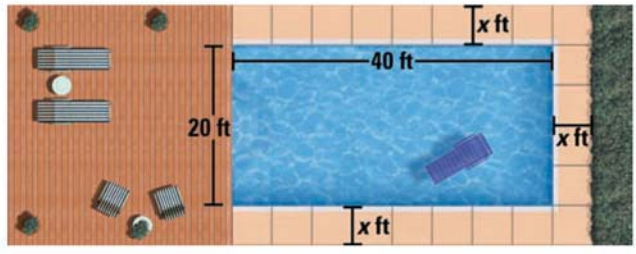
50. SWIMMING POOL A rectangular swiming pool is bordered on one side by a deck. A contractor is hired to build a walkway along the remaining three sides of the pool. The width of the walkway is the same on every side, as shown.



a. Write a polynomial that represents the total area of the pool and the walkway.

b. Find the combined area of the pool and the walkway when the width of the walkway is 5 feet.

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51. SOUND RECORDINGS During the period 1997–2002, the amount of money R (in millions of dollars) spent on sound recordings in the U.S. and the percent P (in decimal form) of this amount spent by people who are between 15 and 19 years old can be modeled by

$$R = -336t^2 + 1730t + 12{,}300 \text{ and } P = 0.00351t^2 - 0.0249t + 0.171$$

where t is the number of years since 1997.

a. Find the values of R and P for $t = 0$. What does the product $R \cdot P$ mean for $t = 0$ in this situation?

b. Write an equation that models the amount spent on sound recordings by people who are between 15 and 19 years old as a function of the number of years since 1997.

c. How much money did people between 15 and 19 years old spend on sound recordings in 2002?

52. ★ SHORT RESPONSE During the period 1980–2002, the number H (in thousands) of housing units in the U.S. and the percent P (in decimal form) of housing units that were vacant can be modeled by

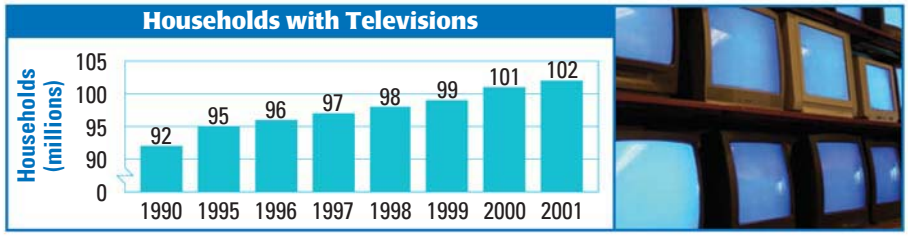
$$H = 1570t + 89{,}000 \quad \text{and} \quad P = 0.0013t + 0.094$$

where t is the number of years since 1980.

a. Write an equation that models the number (in thousands) of vacant housing units as a function of the number of years since 1980. *Explain* how you found this equation.

b. How many housing units were vacant in 2002?

53. ★ **EXTENDED RESPONSE** The bar graph shows the number of households with a television for various years during the period 1990–2001.



a. Find a linear equation that models the number of households T (in millions) with a television as a function of the number of years since 1990. *Explain* how you found your model.

b. During the period 1990–2001, the percent P (in decimal form) of television households that also have a VCR can be modeled by

$$P = -0.0015t^2 + 0.032t + 0.069$$

where t is the number of years since 1990. Write an equation that models the number of households V (in millions) with a VCR and a television as a function of the number of years since 1990.

c. Use the equation from part (b) to predict the number of households that had a VCR and a television in 2002 and in 2005.

54. **CHALLENGE** For the period 1990–2001, the total United States energy consumption C (in quadrillion British Thermal Units, or BTU) and the percent P of the total energy that was consumed in the United States for industrial purposes can be modeled by

$$C = 1.5t + 84$$

$$P = -0.05t^2 + 0.25t + 38$$

where t is the number of years since 1990.

a. Find the percent of total energy that was consumed in the United States for industrial purposes in 2000.

b. Write an equation that gives the total energy (in quadrillion BTU) consumed in the United States for industrial purposes as a function of the number of years since 1990. To write the equation, you may need to rewrite one of the given equations.

MIXED REVIEW

PREVIEW
Prepare for Lesson 9.3 in Exs. 55–60.

Simplify the expression. *(p. 96)*

55. $5(2x - 7) + 5x$

56. $2x + 3(4x - 1)$

57. $15x - 7(x + 3)$

58. $-2x(x + 1) + 2x$

59. $x(x - 4) - 9x$

60. $11x + (x - 1)(8x)$

Solve the system.

61. $2x + y = -5$
$y = -3x + 2$ *(p. 435)*

62. $x - 2y = -7$
$x + 2y = 13$ *(p. 444)*

63. $-2x + 4y = -2$
$x - 2y = -1$ *(p. 451)*

64. $-6x + 4y = 40$
$-3x + 2y = 20$ *(p. 451)*

65. $x \geq -3$
$y < 5$ *(p. 466)*

66. $y \leq 2x - 5$
$y > -3x + 1$ *(p. 466)*

9.3 Find Special Products of Polynomials

Before You multiplied polynomials.

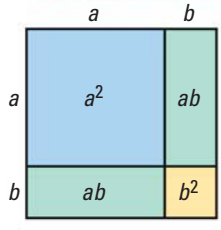
Now You will use special product patterns to multiply polynomials.

Why? So you can make a scientific prediction, as in Example 4.

Key Vocabulary
- **binomial,** *p. 555*
- **trinomial,** *p. 555*

The diagram shows a square with a side length of $(a + b)$ units. You can see that the area of the square is

$$(a + b)^2 = a^2 + 2ab + b^2.$$



This is one version of a pattern called the square of a binomial. To find another version of this pattern, use algebra: replace b with $-b$.

$(a + (-b))^2 = a^2 + 2a(-b) + (-b)^2$ Replace b with $-b$ in the pattern above.

$(a - b)^2 = a^2 - 2ab + b^2$ Simplify.

KEY CONCEPT *For Your Notebook*

Square of a Binomial Pattern

Algebra	Example
$(a + b)^2 = a^2 + 2ab + b^2$	$(x + 5)^2 = x^2 + 10x + 25$
$(a - b)^2 = a^2 - 2ab + b^2$	$(2x - 3)^2 = 4x^2 - 12x + 9$

EXAMPLE 1 Use the square of a binomial pattern

USE PATTERNS
When you use special product patterns, remember that a and b can be numbers, variables, or variable expressions.

Find the product.

a. $(3x + 4)^2 = (3x)^2 + 2(3x)(4) + 4^2$ Square of a binomial pattern

$= 9x^2 + 24x + 16$ Simplify.

b. $(5x - 2y)^2 = (5x)^2 - 2(5x)(2y) + (2y)^2$ Square of a binomial pattern

$= 25x^2 - 20xy + 4y^2$ Simplify.

✓ GUIDED PRACTICE for Example 1

Find the product.

1. $(x + 3)^2$ **2.** $(2x + 1)^2$ **3.** $(4x - y)^2$ **4.** $(3m + n)^2$

SUM AND DIFFERENCE PATTERN To find the product $(x + 2)(x - 2)$, you can multiply the two binomials using the FOIL pattern.

$$(x + 2)(x - 2) = x^2 - 2x + 2x - 4 \qquad \text{Use FOIL pattern.}$$
$$= x^2 - 4 \qquad \text{Combine like terms.}$$

This suggests a pattern for the product of the sum and difference of two terms.

KEY CONCEPT *For Your Notebook*

Sum and Difference Pattern

Algebra	Example
$(a + b)(a - b) = a^2 - b^2$	$(x + 3)(x - 3) = x^2 - 9$

EXAMPLE 2 Use the sum and difference pattern

Find the product.

a. $(t + 5)(t - 5) = t^2 - 5^2$ — Sum and difference pattern

$= t^2 - 25$ — Simplify.

b. $(3x + y)(3x - y) = (3x)^2 - y^2$ — Sum and difference pattern

$= 9x^2 - y^2$ — Simplify.

GUIDED PRACTICE for Example 2

Find the product.

5. $(x + 10)(x - 10)$ **6.** $(2x + 1)(2x - 1)$ **7.** $(x + 3y)(x - 3y)$

SPECIAL PRODUCTS AND MENTAL MATH The special product patterns can help you use mental math to find certain products of numbers.

EXAMPLE 3 Use special products and mental math

Use special products to find the product 26 • 34.

Solution

Notice that 26 is 4 less than 30 while 34 is 4 more than 30.

$26 \cdot 34 = (30 - 4)(30 + 4)$ — Write as product of difference and sum.

$= 30^2 - 4^2$ — Sum and difference pattern

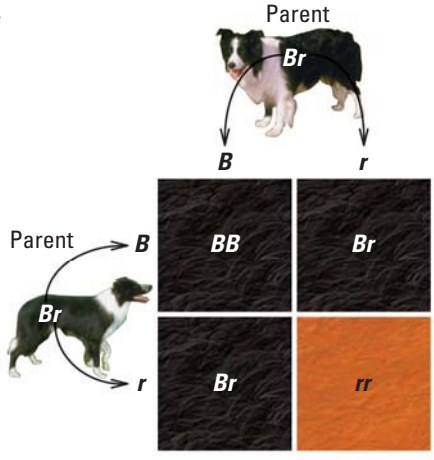
$= 900 - 16$ — Evaluate powers.

$= 884$ — Simplify.

EXAMPLE 4 Solve a multi-step problem

BORDER COLLIES The color of the dark patches of a border collie's coat is determined by a combination of two genes. An offspring inherits one patch color gene from each parent. Each parent has two color genes, and the offspring has an equal chance of inheriting either one.

The gene B is for black patches, and the gene r is for red patches. Any gene combination with a B results in black patches. Suppose each parent has the same gene combination Br. The Punnett square shows the possible gene combinations of the offspring and the resulting patch color.



- What percent of the possible gene combinations of the offspring result in black patches?
- Show how you could use a polynomial to model the possible gene combinations of the offspring.

Solution

STEP 1 **Notice** that the Punnett square shows 4 possible gene combinations of the offspring. Of these combinations, 3 result in black patches.

▶ 75% of the possible gene combinations result in black patches.

STEP 2 **Model** the gene from each parent with $0.5B + 0.5r$. There is an equal chance that the collie inherits a black or red gene from each parent.

The possible genes of the offspring can be modeled by $(0.5B + 0.5r)^2$. Notice that this product also represents the area of the Punnett square.

Expand the product to find the possible patch colors of the offspring.

$$(0.5B + 0.5r)^2 = (0.5B)^2 + 2(0.5B)(0.5r) + (0.5r)^2$$
$$= 0.25B^2 + 0.5Br + 0.25r^2$$

Consider the coefficients in the polynomial.

$$0.25B^2 + 0.5Br + 0.25r^2$$

25% BB, black patches — 50% Br, black patches — 25% rr, red patches

The coefficients show that 25% + 50% = 75% of the possible gene combinations will result in black patches.

GUIDED PRACTICE for Examples 3 and 4

8. *Describe* how you can use special products to find 21^2.

9. **BORDER COLLIES** Look back at Example 4. What percent of the possible gene combinations of the offspring result in red patches?

9.3 EXERCISES

HOMEWORK KEY

- ○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11 and 41
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 18, 42, and 44
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 41

SKILL PRACTICE

1. **VOCABULARY** Give an example of two binomials whose product you can find using the sum and difference pattern.

2. ★ **WRITING** *Explain* how to use the square of a binomial pattern.

EXAMPLE 1 on p. 569 for Exs. 3–10, 18

SQUARE OF A BINOMIAL **Find the product.**

3. $(x + 8)^2$
4. $(a + 6)^2$
5. $(2y + 5)^2$
6. $(t - 7)^2$
7. $(n - 11)^2$
8. $(6b - 1)^2$

ERROR ANALYSIS ***Describe*** **and correct the error in multiplying.**

9. $(s - 3)^2 = s^2 + 9$ ✗

10. $(2d - 10)^2 = 4d^2 - 20d + 100$ ✗

EXAMPLE 2 on p. 570 for Exs. 11–17

SUM AND DIFFERENCE PATTERN **Find the product.**

11. $(t + 4)(t - 4)$
12. $(m - 6)(m + 6)$
13. $(2x + 1)(2x - 1)$
14. $(3x - 1)(3x + 1)$
15. $(7 + w)(7 - w)$
16. $(3s - 8)(3s + 8)$

17. ★ **MULTIPLE CHOICE** Find the product $(7x + 3)(7x - 3)$.

Ⓐ $7x^2 - 9$ Ⓑ $49x^2 - 9$ Ⓒ $49x^2 - 21x - 9$ Ⓓ $49x^2 - 42x - 9$

18. ★ **MULTIPLE CHOICE** Find the product $(5n - 3)^2$.

Ⓐ $5n^2 - 9$ Ⓑ $25n^2 - 9$ Ⓒ $25n^2 - 15n + 9$ Ⓓ $25n^2 - 30n + 9$

EXAMPLE 3 on p. 570 for Exs. 19–22

MENTAL MATH ***Describe*** **how you can use mental math to find the product.**

19. $16 \cdot 24$
20. $28 \cdot 32$
21. 17^2
22. 44^2

SPECIAL PRODUCT PATTERNS **Find the product.**

23. $(r + 9s)^2$
24. $(6x + 5)^2$
25. $(3m + 11n)(3m - 11n)$
26. $(7a + 8b)(7a - 8b)$
27. $(3m - 7n)^2$
28. $(13 - 2x)^2$
29. $(3f - 9)(3f + 9)$
30. $(9 - 4t)(9 + 4t)$
31. $(3x + 8y)^2$
32. $(-x - 2y)^2$
33. $(2a - 5b)(2a + 5b)$
34. $(6x + y)(6x - y)$

MULTIPLYING FUNCTIONS **Perform the indicated operation using the functions $f(x) = 3x + 0.5$ and $g(x) = 3x - 0.5$.**

35. $f(x) \cdot g(x)$
36. $(f(x))^2$
37. $(g(x))^2$

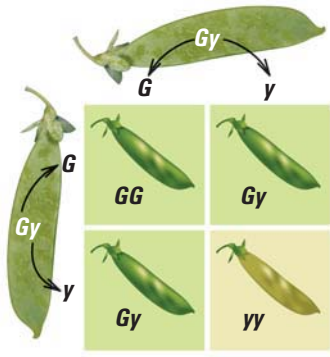
38. **CHALLENGE** Write two binomials that have the product $x^2 - 121$. *Explain*.

39. **CHALLENGE** Write a pattern for the cube of a binomial $(a + b)^3$.

PROBLEM SOLVING

EXAMPLE 4 on p. 571 for Exs. 40–42

40. PEA PLANTS In pea plants, the gene G is for green pods, and the gene y is for yellow pods. Any gene combination with a G results in a green pod. Suppose two pea plants have the same gene combination Gy. The Punnett square shows the possible gene combinations of an offspring pea plant and the resulting pod color.



a. What percent of possible gene combinations of the offspring plant result in a yellow pod?

b. Show how you could use a polynomial to model the possible gene combinations of the offspring.

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41. MULTIPLE REPRESENTATIONS In humans, the gene s is for straight thumbs, and the gene C is for curved thumbs. Any gene combination with a C results in a curved thumb. Suppose each parent has the same gene combination Cs.

a. Making a Diagram Make a Punnet square that shows the possible gene combinations inherited by a child.

b. Writing a Model Write a polynomial that models the possible gene combinations of the child.

c. Interpreting a Model What percent of the possible gene combinations of the child result in a curved thumb?

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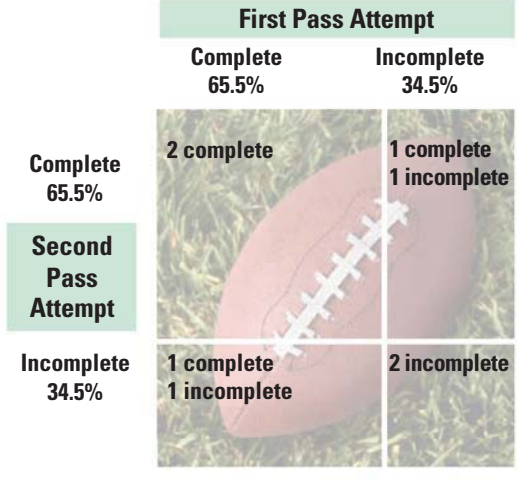
42. ★ SHORT RESPONSE In ball pythons, the gene N is for normal coloring, and the gene a is for no coloring, or albino. Any gene combination with an N results in normal coloring. Suppose one parent python has the gene combination Na and the other parent python has the gene combination aa. What percent of the possible gene combinations of the offspring result in an albino python? *Explain* how you found your answer.

43. FOOTBALL STATISTICS During the 2004 regular season, the San Diego Chargers' quarterback Drew Brees completed 65.5% of the passes he attempted. The area model shows the possible outcomes of two attempted passes.



a. What percent of the possible outcomes of two attempted passes results in Drew Brees's throwing at least one complete pass? *Explain* how you found your answer using the area model.

b. Show how you could use a polynomial to model the possible results of two attempted passes.

44. ★ **EXTENDED RESPONSE** The iris of an eye surrounds the pupil. It regulates the amount of light entering the eye by opening and closing the pupil. For parts (a)–(c) below, leave your answers in terms of π.

The iris of a human eye has a width w that varies from 0.5 millimeter to 4 millimeters.

a. Write a polynomial that represents the pupil's radius.

b. Write a polynomial that represents the pupil's area.

c. What is the least possible area and the greatest possible area of the pupil? *Explain* how you found your answers.

45. **CHALLENGE** You use 100 feet of fencing to form a square with a side length of 25 feet. You want to change the dimensions of the enclosed region. For every 1 foot you increase the width, you must decrease the length by 1 foot. Write a polynomial that gives the area of the rectangle after you increase the width by x feet and decrease the length by x feet. *Explain* why *any* change in dimensions results in an area less than that of the original square.

MIXED REVIEW

PREVIEW
Prepare for Lesson 9.4 in Exs. 46–53.

Find the greatest common factor of the pair of numbers. *(p. 910)*

46. 25, 30 **47.** 36, 54 **48.** 14, 21 **49.** 36, 50

50. 65, 39 **51.** 13, 20 **52.** 77, 143 **53.** 24, 162

Solve the equation. Check your solution.

54. $x + 11 = 6$ *(p. 134)* **55.** $11x + 8 = -14$ *(p. 141)*

56. $2x - 5(x - 13) = 35$ *(p. 148)* **57.** $9x + 4 - 4x = 6x + 7$ *(p. 154)*

QUIZ *for Lessons 9.1–9.3*

Find the sum, difference, or product.

1. $(x^2 - 3x + 5) + (-2x^2 + 11x + 1)$ *(p. 554)* **2.** $(8y^3 - 7y^2 + y) - (9y^2 - 5y + 7)$ *(p. 554)*

3. $(2r + 11)(r - 6)$ *(p. 562)* **4.** $(m + 3)(-2m^2 + 5m - 1)$ *(p. 562)*

5. $(2 + 8p)(2 - 10p)$ *(p. 562)* **6.** $(15 - 2s)^2$ *(p. 569)*

7. $(5w + 9z)^2$ *(p. 569)* **8.** $(5x - 4y)(5x + 4y)$ *(p. 569)*

9. **AREA** The length of a rectangular rug is 2 times its width. The rug is centered in a rectangular room. Each edge is 3 feet from the nearest wall. Write a polynomial that represents the area of the room. *(p. 564)*

9.4 Solve Polynomial Equations in Factored Form

Before You solved linear equations.

Now You will solve polynomial equations.

Why So you can analyze vertical motion, as in Ex. 55.

Key Vocabulary
- **roots**
- **vertical motion model**

In Lesson 2.4, you learned the property of zero: For any real number a, $a \cdot 0 = 0$. This is equivalent to saying:

For real numbers a and b, if $a = 0$ or $b = 0$, then $ab = 0$.

The converse of this statement is also true (as shown in Exercise 49), and it is called the zero-product property.

KEY CONCEPT *For Your Notebook*

Zero-Product Property

Let a and b be real numbers. If $ab = 0$, then $a = 0$ or $b = 0$.

The zero-product property is used to solve an equation when one side is zero and the other side is a product of polynomial factors. The solutions of such an equation are also called **roots**.

EXAMPLE 1 Use the zero-product property

Solve $(x - 4)(x + 2) = 0$.

$(x - 4)(x + 2) = 0$ Write original equation.

$x - 4 = 0$ *or* $x + 2 = 0$ Zero-product property

$x = 4$ *or* $x = -2$ Solve for x.

▶ The solutions of the equation are 4 and -2.

CHECK Substitute each solution into the original equation to check.

$(4 - 4)(4 + 2) \stackrel{?}{=} 0$ $\quad$ $(-2 - 4)(-2 + 2) \stackrel{?}{=} 0$

$0 \cdot 6 \stackrel{?}{=} 0$ $\quad$ $-6 \cdot 0 \stackrel{?}{=} 0$

$0 = 0$ ✓ $\quad$ $0 = 0$ ✓

GUIDED PRACTICE for Example 1

1. Solve the equation $(x - 5)(x - 1) = 0$.

REVIEW GCF
For help with finding the GCF, see p. 910.

FACTORING To solve a polynomial equation using the zero-product property, you may need to *factor* the polynomial, or write it as a product of other polynomials. Look for the *greatest common factor* (GCF) of the polynomial's terms. This is a monomial with an integer coefficient that divides evenly into each term.

EXAMPLE 2 Find the greatest common monomial factor

Factor out the greatest common monomial factor.

a. $12x + 42y$ **b.** $4x^4 + 24x^3$

Solution

a. The GCF of 12 and 42 is 6. The variables x and y have no common factor. So, the greatest common monomial factor of the terms is 6.

▶ $12x + 42y = 6(2x + 7y)$

b. The GCF of 4 and 24 is 4. The GCF of x^4 and x^3 is x^3. So, the greatest common monomial factor of the terms is $4x^3$.

▶ $4x^4 + 24x^3 = 4x^3(x + 6)$

GUIDED PRACTICE for Example 2

2. Factor out the greatest common monomial factor from $14m + 35n$.

EXAMPLE 3 Solve an equation by factoring

Solve $2x^2 + 8x = 0$.

$2x^2 + 8x = 0$	Write original equation.
$2x(x + 4) = 0$	Factor left side.
$2x = 0$ *or* $x + 4 = 0$	Zero-product property
$x = 0$ *or* $x = -4$	Solve for x.

▶ The solutions of the equation are 0 and -4.

EXAMPLE 4 Solve an equation by factoring

Solve $6n^2 = 15n$.

AVOID ERRORS
To use the zero-product property, you must write the equation so that one side is 0. For this reason, $15n$ must be subtracted from each side.

$6n^2 - 15n = 0$	Subtract $15n$ from each side.
$3n(2n - 5) = 0$	Factor left side.
$3n = 0$ *or* $2n - 5 = 0$	Zero-product property
$n = 0$ *or* $n = \frac{5}{2}$	Solve for n.

▶ The solutions of the equation are 0 and $\frac{5}{2}$.

GUIDED PRACTICE for Examples 3 and 4

Solve the equation.

3. $a^2 + 5a = 0$ **4.** $3s^2 - 9s = 0$ **5.** $4x^2 = 2x$

VERTICAL MOTION A *projectile* is an object that is propelled into the air but has no power to keep itself in the air. A thrown ball is a projectile, but an airplane is not. The height of a projectile can be described by the **vertical motion model**.

UNDERSTAND THE MODEL
The vertical motion model takes into account the effect of gravity but ignores other, less significant, factors such as air resistance.

KEY CONCEPT *For Your Notebook*

Vertical Motion Model

The height h (in feet) of a projectile can be modeled by

$$h = -16t^2 + vt + s$$

where t is the time (in seconds) the object has been in the air, v is the initial vertical velocity (in feet per second), and s is the initial height (in feet).

EXAMPLE 5 Solve a multi-step problem

ARMADILLO A startled armadillo jumps straight into the air with an initial vertical velocity of 14 feet per second. After how many seconds does it land on the ground?

Solution

STEP 1 **Write** a model for the armadillo's height above the ground.

$h = -16t^2 + vt + s$ Vertical motion model

$h = -16t^2 + 14t + 0$ Substitute 14 for v and 0 for s.

$h = -16t^2 + 14t$ Simplify.

STEP 2 **Substitute** 0 for h. When the armadillo lands, its height above the ground is 0 feet. Solve for t.

$0 = -16t^2 + 14t$ Substitute 0 for h.

$0 = 2t(-8t + 7)$ Factor right side.

$2t = 0$ *or* $-8t + 7 = 0$ Zero-product property

$t = 0$ *or* $t = 0.875$ Solve for t.

AVOID ERRORS
The solution $t = 0$ means that before the armadillo jumps, its height above the ground is 0 feet.

▶ The armadillo lands on the ground 0.875 second after the armadillo jumps.

GUIDED PRACTICE for Example 5

6. WHAT IF? In Example 5, suppose the initial vertical velocity is 12 feet per second. After how many seconds does the armadillo land on the ground?

9.4 EXERCISES

HOMEWORK KEY

- = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3 and 55
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 39, 53, and 56
- = **MULTIPLE REPRESENTATIONS** Ex. 58

SKILL PRACTICE

1. **VOCABULARY** What is the vertical motion model and what does each variable in the model represent?

2. ★ **WRITING** *Explain* how to use the zero-product property to find the solutions of the equation $3x(x - 7) = 0$.

EXAMPLE 1 on p. 575 for Exs. 3–16

ZERO-PRODUCT PROPERTY **Solve the equation.**

3. $(x - 5)(x + 3) = 0$
4. $(y + 9)(y - 1) = 0$
5. $(z - 13)(z - 14) = 0$
6. $(c + 6)(c + 8) = 0$
7. $(d - 7)\left(d + \frac{4}{3}\right) = 0$
8. $\left(g - \frac{1}{8}\right)(g + 18) = 0$
9. $(m - 3)(4m + 12) = 0$
10. $(2n - 14)(3n + 9) = 0$
11. $(3n + 11)(n + 1) = 0$
12. $(3x + 1)(x + 6) = 0$
13. $(2y + 5)(7y - 5) = 0$
14. $(8z - 6)(12z + 14) = 0$

15. ★ **MULTIPLE CHOICE** What are the solutions of the equation $(y - 12)(y + 6) = 0$?

Ⓐ -12 and -6 Ⓑ -12 and 6 Ⓒ -6 and 12 Ⓓ 6 and 12

16. **ERROR ANALYSIS** *Describe* and correct the error in solving $(z - 15)(z + 21) = 0$.

$(z - 15)(z + 21) = 0$
$z = -15$ or $z = 21$

EXAMPLE 2 on p. 576 for Exs. 17–26

FACTORING EXPRESSIONS **Factor out the greatest common monomial factor.**

17. $2x + 2y$
18. $6x^2 - 15y$
19. $3s^4 + 16s$
20. $5d^6 + 2d^5$
21. $7w^5 - 35w^2$
22. $9m^7 - 3m^2$
23. $15n^3 + 25n$
24. $12a^5 + 8a$
25. $\frac{5}{2}x^6 - \frac{1}{2}x^4$

26. **ERROR ANALYSIS** *Describe* and correct the error in factoring out the greatest common monomial factor of $18x^8 - 9x^4 - 6x^3$.

$18x^8 - 9x^4 - 6x^3 = 3x(6x^7 - 3x^3 - 2x^2)$

EXAMPLES 3 and 4 on p. 576 for Exs. 27–39

SOLVING EQUATIONS **Solve the equation.**

27. $b^2 + 6b = 0$
28. $5w^2 - 5w = 0$
29. $-10n^2 + 35n = 0$
30. $2x^2 + 15x = 0$
31. $18c^2 + 6c = 0$
32. $-32y^2 - 24y = 0$
33. $3k^2 = 6k$
34. $6h^2 = 3h$
35. $4s^2 = 10s$
36. $-42z^2 = 14z$
37. $28m^2 = -8m$
38. $-12p^2 = -30p$

39. ★ **MULTIPLE CHOICE** What are the solutions of $4x^2 = x$?

Ⓐ -4 and 0 Ⓑ $-\frac{1}{4}$ and 0 Ⓒ 0 and $\frac{1}{4}$ Ⓓ 0 and 4

FACTORING EXPRESSIONS **Factor out the greatest common monomial factor.**

40. $20x^2y^2 - 4xy$ **41.** $8a^2b - 6ab^2$ **42.** $18s^2t^5 - 2s^3t$

43. $v^3 - 5v^2 + 9v$ **44.** $-2g^4 + 14g^2 + 6g$ **45.** $6q^5 - 21q^4 - 15q^2$

HINT
For help with finding zeros of functions, see p. 335.

FINDING ZEROS OF FUNCTIONS **Find the zeros of the function.**

46. $f(x) = x^2 - 15x$ **47.** $f(x) = -2x^2 + x$ **48.** $f(x) = 3x^2 - 27x$

49. **CHALLENGE** Consider the equation $ab = 0$. Assume that $a \neq 0$ and solve the equation for b. Then assume that $b \neq 0$ and solve the equation for a. What conclusion can you draw about the values of a and b?

50. **CHALLENGE** Consider the equation $z = x^2 - xy$. For what values of x and y does $z = 0$?

PROBLEM SOLVING

EXAMPLE 5
on p. 577
for Exs. 51–53

51. **MOTION** A cat leaps from the ground into the air with an initial vertical velocity of 11 feet per second. After how many seconds does the cat land on the ground?

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52. **SPITTLEBUG** A spittlebug jumps into the air with an initial vertical velocity of 10 feet per second.

a. Write an equation that gives the height of the spittlebug as a function of the time (in seconds) since it left the ground.

b. The spittlebug reaches its maximum height after 0.3125 second. How high can it jump?

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53. ★ **SHORT RESPONSE** A penguin jumps out of the water while swimming. This action is called porpoising. The height h (in feet) of the porpoising penguin can be modeled by $h = -16t^2 + 4.5t$ where t is the time (in seconds) since the penguin jumped out of the water. Find the zeros of the function. *Explain* what the zeros mean in this situation.

VERTICAL MOTION **In Exercises 54 and 55, use the information below.**

The height h (in meters) of a projectile can be modeled by $h = -4.9t^2 + vt + s$ where t is the time (in seconds) the object has been in the air, v is the initial vertical velocity (in meters per second), and s is the initial height (in meters).

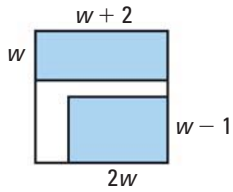
54. **SOCCER** A soccer ball is kicked upward from the ground with an initial vertical velocity of 3.6 meters per second. After how many seconds does it land?

55. **RABBIT HIGH JUMP** A rabbit in a high jump competition leaves the ground with an initial vertical velocity of 4.9 meters per second.

a. Write an equation that gives the height of the rabbit as a function of the time (in seconds) since it left the ground.

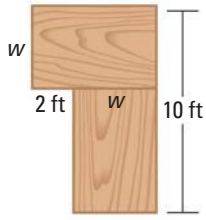
b. What is a reasonable domain for the function? *Explain* your answer.

56. ★ **MULTIPLE CHOICE** Two rectangular rooms in a building's floor plan have different dimensions but the same area. The dimensions (in meters) are shown. What is the value of w?



Ⓐ 3 m Ⓑ 4 m Ⓒ 6 m Ⓓ 8 m

57. **TABLETOP AREAS** A display in your school library sits on top of two rectangular tables arranged in an L shape, as shown. The tabletops have the same area.



a. Write an equation that relates the areas of the tabletops.

b. Find the value of w.

c. What is the combined area of the tabletops?

58. **MULTIPLE REPRESENTATIONS** An arch frames the entrance to a garden. The shape of the arch is modeled by the graph of the equation $y = -2x^2 + 8x$ where x and y are measured in feet. On a coordinate plane, the ground is represented by the x-axis.

a. Making a Table Make a table of values that shows the height of the arch for $x = 0$, 1, 2, 3, and 4 feet.

b. Drawing a Graph Plot the ordered pairs in the table as points in a coordinate plane. Connect the points with a smooth curve that represents the arch.

c. Interpreting a Graph How wide is the base of the arch?

59. **CHALLENGE** The shape of an arched doorway is modeled by the graph of the function $y = -0.5x(x - 8)$ where x and y are measured in feet. On a coordinate plane, the floor is represented by the x-axis.

a. How wide is the doorway at its base? *Justify* your answer using the zeros of the function.

b. The doorway's highest point occurs above the center of its base. How high is the highest point of the arched doorway? *Explain* how you found your answer.

MIXED REVIEW

PREVIEW
Prepare for Lesson 9.5 in Exs. 60–71.

Find the product.

60. $45(-x)(-x)$ *(p. 88)*

61. $-9a(-6a)(-a)$ *(p. 88)*

62. $-7(8n)(-4)$ *(p. 88)*

63. $(y - 1)(y + 7)$ *(p. 562)*

64. $(m - 5)(m - 13)$ *(p. 562)*

65. $(2b + 5)(b + 3)$ *(p. 562)*

66. $(3p + 8)(4p - 1)$ *(p. 562)*

67. $(5z - 2)(5z - 4)$ *(p. 562)*

68. $(9t + 7)(4t + 5)$ *(p. 562)*

69. $(2c + 7)^2$ *(p. 569)*

70. $(9 - 5w)^2$ *(p. 569)*

71. $(3g - 4h)^2$ *(p. 569)*

Graph the system of linear inequalities. *(p. 466)*

72. $x > -3$
$x \leq 3$

73. $x \geq 0$
$-3x + y < -1$
$y \geq 0$

74. $x < 6$
$y > -4$
$y < 2$
$y \leq x$

MIXED REVIEW of Problem Solving

STATE TEST PRACTICE
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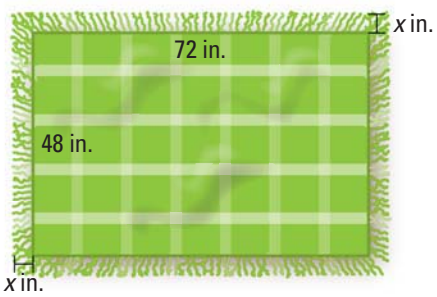
Lessons 9.1–9.4

1. MULTI-STEP PROBLEM You are making a blanket with a fringe border of equal width on each edge, as shown.



a. Write a polynomial that represents the total area of the blanket with the fringe.

b. Find the total area of the blanket with fringe when the width of the fringe is 4 inches.

2. OPEN-ENDED A horse with pinto coloring has white fur with patches of color. The gene P is for pinto coloring, and the gene s is for solid coloring. Any gene combination with a P results in pinto coloring.

a. Suppose a male horse has the gene combination Ps. Choose a color gene combination for a female horse. Create a Punnett square to show the possible gene combinations of the two horses' offspring.

b. What percent of the possible gene combinations of the offspring result in pinto coloring?

c. Show how you could use a polynomial to model the possible color gene combinations of the offspring.

3. SHORT RESPONSE One football is kicked into the air with an initial vertical velocity of 44 feet per second. Another football is kicked into the air with an initial vertical velocity of 40 feet per second.

a. Which football is in the air for more time?

b. *Justify* your answer to part (a).

4. GRIDDED ANSWER During the period 1996–2000, the total value T (in millions of dollars) of toys imported to the United States can be modeled by

$$T = 82.9t^3 - 848t^2 + 3030t + 9610$$

where t is the number of years since 1996. What is the degree of the polynomial that represents T?

5. EXTENDED RESPONSE During the period 1992–2000, the number C (in millions) of people participating in cross-country skiing and the number S (in millions) of people participating in snowboarding can be modeled by

$$C = 0.067t^3 - 0.107t^2 + 0.27t + 3.5$$

$$S = 0.416t + 1.24$$

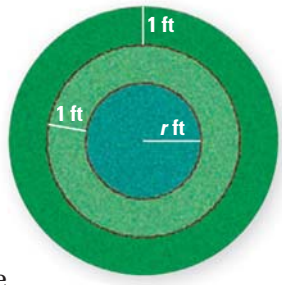
where t is the number of years since 1992.

a. Write an equation that models the total number of people T (in millions) participating in cross-country skiing and snowboarding as a function of the number of years since 1992.

b. Find the total participation in these activities in 1992 and 2000.

c. What was the average rate of change in total participation from 1992 to 2000? *Explain* how you found this rate.

6. SHORT RESPONSE A circular rug has an interior circle and two rings around the circle, as shown.



a. Write a polynomial that represents the total area of the rug. Leave your answer in terms of π.

b. The interior circle of the rug has a diameter of 3 feet. What is the area of the rug? Leave your answer in terms of π. *Explain* how you found your answer.

9.5 Factorization with Algebra Tiles

MATERIALS • algebra tiles

QUESTION How can you factor a trinomial using algebra tiles?

You have seen that algebra tiles can be used to model polynomials and to multiply binomials. Now, you will use algebra tiles to factor trinomials.

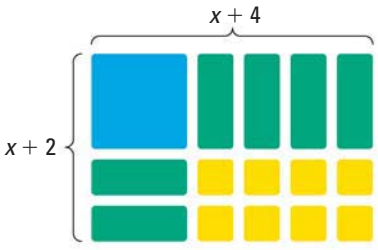
EXPLORE Factor the trinomial $x^2 + 6x + 8$

STEP 1 Make a rectangle

Model the trinomial with algebra tiles. You will need one x^2-tile, six x-tiles, and eight 1-tiles. Arrange all of the tiles to form a rectangle. There can be no gaps or leftover tiles. The area of the rectangle represents the trinomial.

STEP 2 Find the side lengths

The side lengths of the rectangle represent the polynomials $x + 2$ and $x + 4$. So, $x^2 + 6x + 8 = (x + 2)(x + 4)$.



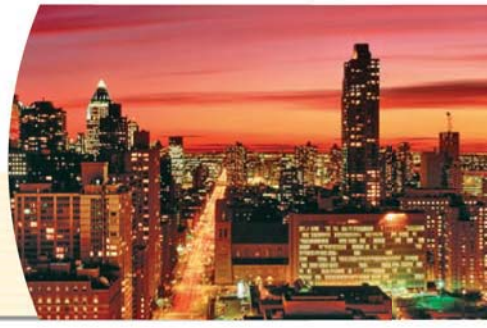
DRAW CONCLUSIONS Use your observations to complete these exercises

1. Use multiplication to show that $x + 4$ and $x + 2$ are factors of the polynomial $x^2 + 6x + 8$.

Use algebra tiles to factor the trinomial. Include a drawing of your model.

2. $x^2 + 6x + 5$
3. $x^2 + 9x + 14$
4. $x^2 + 5x + 6$
5. $x^2 + 8x + 16$
6. $x^2 + 5x + 4$
7. $x^2 + 8x + 12$
8. **REASONING** The factors of the trinomial $x^2 + 6x + 8$ have the form $x + p$ and $x + q$, as shown above. How are p and q related to 6 and 8?

9.5 Factor $x^2 + bx + c$



Before You factored out the greatest common monomial factor.

Now You will factor trinomials of the form $x^2 + bx + c$.

Why So you can find the dimensions of figures, as in Ex. 61.

Key Vocabulary
- **zero of a function,** *p. 337*

From Lesson 9.2, you know that

$$(x + 3)(x + 4) = x^2 + (4 + 3)x + 4 \cdot 3 = x^2 + 7x + 12.$$

You will reverse this process to factor trinomials of the form $x^2 + bx + c$.

KEY CONCEPT *For Your Notebook*

Factoring $x^2 + bx + c$

Algebra $x^2 + bx + c = (x + p)(x + q)$ provided $p + q = b$ and $pq = c$.

Example $x^2 + 5x + 6 = (x + 3)(x + 2)$ because $3 + 2 = 5$ and $3 \cdot 2 = 6$.

EXAMPLE 1 Factor when b and c are positive

Factor $x^2 + 11x + 18$.

Solution

Find two positive factors of 18 whose sum is 11. Make an organized list.

Factors of 18	Sum of factors	
18, 1	$18 + 1 = 19$	✗
9, 2	$9 + 2 = \mathbf{11}$	← Correct sum
6, 3	$6 + 3 = 9$	✗

The factors 9 and 2 have a sum of 11, so they are the correct values of p and q.

▶ $x^2 + 11x + 18 = (x + \mathbf{9})(x + \mathbf{2})$

CHECK $(x + 9)(x + 2) = x^2 + 2x + 9x + 18$ Multiply binomials.

$= x^2 + 11x + 18$ ✓ Simplify.

GUIDED PRACTICE for Example 1

Factor the trinomial.

1. $x^2 + 3x + 2$ **2.** $a^2 + 7a + 10$ **3.** $t^2 + 9t + 14$

FACTORING When factoring a trinomial, first consider the signs of p and q.

$(x + p)(x + q)$	$x^2 + bx + c$	Signs of b and c
$(x + 2)(x + 3)$	$x^2 + 5x + 6$	b is positive; c is positive.
$(x + 2)(x + (-3))$	$x^2 - x - 6$	b is negative; c is negative.
$(x + (-2))(x + 3)$	$x^2 + x - 6$	b is positive; c is negative.
$(x + (-2))(x + (-3))$	$x^2 - 5x + 6$	b is negative; c is positive.

By observing the signs of b and c in the table, you can see that:

- b and c are positive when both p and q are positive.
- b is negative and c is positive when both p and q are negative.
- c is negative when p and q have different signs.

EXAMPLE 2 Factor when b is negative and c is positive

Factor $n^2 - 6n + 8$.

Because b is negative and c is positive, p and q must both be negative.

Factors of 8	Sum of factors	
$-8, -1$	$-8 + (-1) = -9$	✗
$-4, -2$	$-4 + (-2) = -6$	← Correct sum

▶ $n^2 - 6n + 8 = (n - 4)(n - 2)$

EXAMPLE 3 Factor when b is positive and c is negative

Factor $y^2 + 2y - 15$.

Because c is negative, p and q must have different signs.

Factors of −15	Sum of factors	
$-15, 1$	$-15 + 1 = -14$	✗
$15, -1$	$15 + (-1) = 14$	✗
$-5, 3$	$-5 + 3 = -2$	✗
$5, -3$	$5 + (-3) = 2$	← Correct sum

▶ $y^2 + 2y - 15 = (y + 5)(y - 3)$

GUIDED PRACTICE for Examples 2 and 3

Factor the trinomial.

4. $x^2 - 4x + 3$ **5.** $t^2 - 8t + 12$ **6.** $m^2 + m - 20$ **7.** $w^2 + 6w - 16$

EXAMPLE 4 Solve a polynomial equation

Solve the equation $x^2 + 3x = 18$.

$x^2 + 3x = 18$ — Write original equation.

$x^2 + 3x - 18 = 0$ — Subtract 18 from each side.

$(x + 6)(x - 3) = 0$ — Factor left side.

$x + 6 = 0$ *or* $x - 3 = 0$ — Zero-product property

$x = -6$ *or* $x = 3$ — Solve for x.

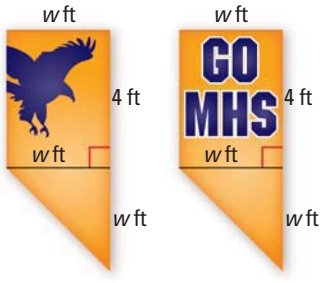
▶ The solutions of the equation are -6 and 3.

GUIDED PRACTICE for Example 4

8. Solve the equation $s^2 - 2s = 24$.

EXAMPLE 5 Solve a multi-step problem

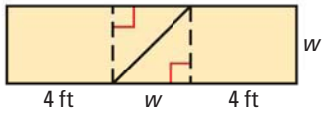
BANNER DIMENSIONS You are making banners to hang during school spirit week. Each banner requires 16.5 square feet of felt and will be cut as shown. Find the width of one banner.



ANOTHER WAY
For alternative methods for solving Example 5, turn to page 590 for the **Problem Solving Workshop**.

Solution

STEP 1 **Draw** a diagram of two banners together.



STEP 2 **Write** an equation using the fact that the area of 2 banners is $2(16.5) = 33$ square feet. Solve the equation for w.

$A = \ell \cdot w$ — Formula for area of a rectangle

$33 = (4 + w + 4) \cdot w$ — Substitute 33 for A and $(4 + w + 4)$ for ℓ.

$0 = w^2 + 8w - 33$ — Simplify and subtract 33 from each side.

$0 = (w + 11)(w - 3)$ — Factor right side.

$w + 11 = 0$ *or* $w - 3 = 0$ — Zero-product property

$w = -11$ *or* $w = 3$ — Solve for w.

▶ The banner cannot have a negative width, so the width is 3 feet.

GUIDED PRACTICE for Example 5

9. WHAT IF? In Example 5, suppose the area of a banner is to be 10 square feet. What is the width of one banner?

9.5 EXERCISES

HOMEWORK KEY

- = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7 and 61
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 29, 42, 61, 62, and 63
- = **MULTIPLE REPRESENTATIONS** Ex. 64

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The ___?___ of $t^2 + 2t + 3$ are $t + 2$ and $t + 1$.

2. ★ **WRITING** If $x^2 - 8x + 12 = (x + p)(x + q)$, what are the signs of p and q? *Justify* your answer.

EXAMPLES 1, 2, and 3 on pp. 583–584 for Exs. 3–19

FACTORING TRINOMIALS **Factor the trinomial.**

3. $x^2 + 4x + 3$
4. $a^2 + 6a + 8$
5. $b^2 - 17b + 72$
6. $s^2 - 10s + 16$
7. $z^2 + 8z - 48$
8. $w^2 + 18w + 56$
9. $y^2 - 7y - 18$
10. $n^2 - 9n + 14$
11. $x^2 + 3x - 70$
12. $f^2 + 4f - 32$
13. $m^2 - 7m - 120$
14. $d^2 - 20d + 99$
15. $p^2 + 20p + 64$
16. $x^2 + 6x - 72$
17. $c^2 + 15c + 44$

ERROR ANALYSIS ***Describe*** **and correct the error in factoring the trinomial.**

18. $s^2 - 17s - 60 = (s - 5)(s - 12)$ ✗

19. $m^2 - 10m + 24 = (m - 12)(m + 2)$ ✗

EXAMPLE 4 on p. 585 for Exs. 20–29

SOLVING EQUATIONS **Solve the equation.**

20. $x^2 - 10x + 21 = 0$
21. $n^2 - 7n - 30 = 0$
22. $w^2 - 15w + 44 = 0$
23. $a^2 + 5a = 50$
24. $r^2 + 2r = 24$
25. $t^2 + 9t = -20$
26. $y^2 - 2y - 8 = 7$
27. $m^2 + 22 = -23m$
28. $b^2 + 5 = 8b - 10$

29. ★ **MULTIPLE CHOICE** What are the solutions of the equation $x^2 - 8x = 240$?

(A) -20 and -12 (B) -20 and 12

(C) 20 and -12 (D) 12 and 20

FINDING ZEROS OF FUNCTIONS **Find the zeros of the polynomial function.**

30. $f(x) = x^2 + 11x + 18$
31. $g(x) = x^2 + 5x + 6$
32. $h(x) = x^2 - 18x + 32$
33. $f(x) = x^2 - 14x + 45$
34. $h(x) = x^2 - 5x - 24$
35. $g(x) = x^2 - 14x - 51$
36. $g(x) = x^2 + 10x - 39$
37. $f(x) = -x^2 + 16x - 28$
38. $f(x) = -x^2 + 24x + 180$

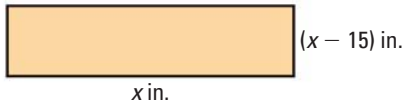
SOLVING EQUATIONS **Solve the equation.**

39. $s(s + 1) = 72$
40. $x^2 - 10(x - 1) = -11$
41. $q(q + 19) = -34$

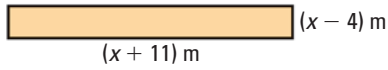
42. ★ **SHORT RESPONSE** Write an equation of the form $x^2 + bx + c = 0$ that has the solutions -4 and 6. *Explain* how you found your answer.

GEOMETRY **Find the dimensions of the rectangle or triangle that has the given area.**

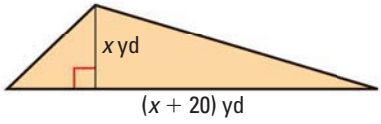
43. Area: 100 square inches



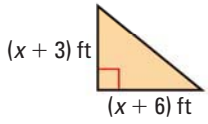
44. Area: 34 square meters



45. Area: 78 square yards



46. Area: 119 square feet



FACTORING TRINOMIALS **In Exercises 47–55, use the example below to factor the trinomial.**

EXAMPLE **Factor a trinomial in two variables**

Factor $x^2 + 9xy + 14y^2$.

Solution

To factor the trinomial, you must find factors of the form $x + py$ and $x + qy$.

First, consider the signs of the factors needed. In this example, b is 9, and c is 14. Because both b and c are positive, you must find two positive factors of 14 that have a sum of 9.

Factors of 14	Sum of factors	
14, 1	14 + 1 = 15	✗
7, 2	7 + 2 = 9	← Correct sum

The factors 7 and 2 have a sum of 9, so 7 and 2 are the correct values of p and q.

▶ $x^2 + 9xy + 14y^2 = (x + 7y)(x + 2y)$

47. $x^2 - 4xy + 4y^2$ **48.** $y^2 - 6yz + 5z^2$ **49.** $c^2 + 13cd + 36d^2$

50. $r^2 + 15rs + 50s^2$ **51.** $a^2 + 2ab - 15b^2$ **52.** $x^2 + 8xy - 65y^2$

53. $m^2 - mn - 42n^2$ **54.** $u^2 - 3uv - 108v^2$ **55.** $g^2 + 4gh - 60h^2$

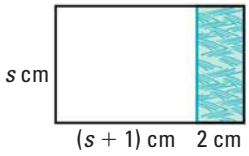
CHALLENGE **Find all integer values of b for which the trinomial has factors of the form $x + p$ and $x + q$ where p and q are integers.**

56. $x^2 + bx + 15$ **57.** $x^2 - bx + 21$ **58.** $x^2 + bx - 42$

PROBLEM SOLVING

EXAMPLE 5 on p. 585 for Exs. 59–61

59. CARD DESIGN You are designing a gift card that has a border along one side, as shown. The area of the white part of the card is 30 square centimeters. What is the area of the border?



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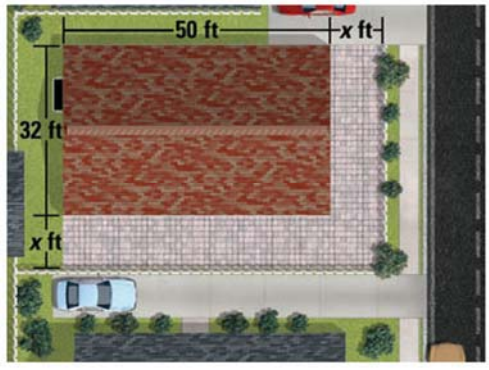
60. CONSTRUCTION A contractor is building a porch along two sides of a house. The house is rectangular with a width of 32 feet and a length of 50 feet. The porch will have the same width on each side of the house.



a. Write a polynomial that represents the combined area of the first floor of the house and the porch.

b. The owners want the combined area of the first floor and the porch to be 2320 square feet. How wide should the contractor build the porch?

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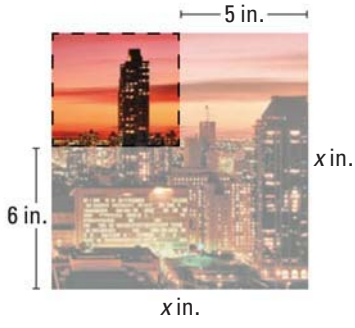
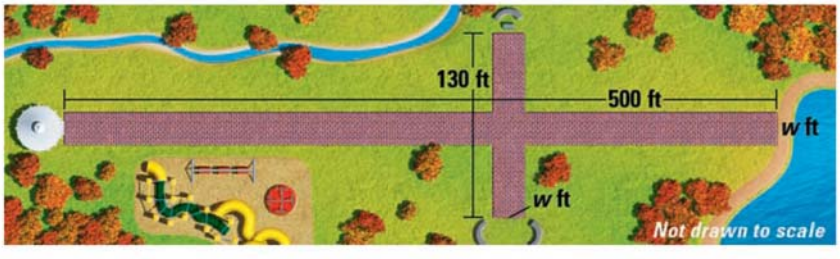
61. ★ **SHORT RESPONSE** You trimmed a large square picture so that you could fit it into a frame. You trimmed 6 inches from the length and 5 inches from the width. The area of the resulting picture is 20 square inches. What was the perimeter of the original large square picture? *Explain* how you found your answer.



62. ★ **EXTENDED RESPONSE** A town has a rectangular park. The parks department is planning to install two brick paths that will intersect at right angles. One path will be 130 feet long, and the other path will be 500 feet long. The paths will have the same width.



HINT Add the path areas, but subtract the overlap, so that it is not counted twice.

a. Write a polynomial that represents the combined area of the two paths.

b. The parks department can afford brick for 3125 square feet of path. Write and solve an equation to find the width of the paths.

c. In part (b) you used one solution of the equation to find your answer. *Explain* how you chose which solution to use.

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE = MULTIPLE REPRESENTATIONS

63. ★ **MULTIPLE CHOICE** A square quilt has a border that is 1 foot wide on each side. The quilt has an area of 25 square feet. What is the side length of the quilt without the border?

Ⓐ 2 feet Ⓑ 3 feet Ⓒ 4 feet Ⓓ 5 feet

64. **MULTIPLE REPRESENTATIONS** You toss a set of keys to a friend who is standing at a window 20 feet above the ground in a building that is 5 feet away from where you are standing. The path of the keys can be modeled by the graph of the equation $y = -x^2 + 8x + 5$ where x and y are measured in feet. On a coordinate plane, the ground is represented by the x-axis, and you are standing at the origin.

a. Making a Table Make a table of values that shows the height of the keys for $x = 2$, 4, 6, and 8 feet.

b. Drawing a Graph Plot the ordered pairs in the table as points in a coordinate plane. Connect the points with a smooth curve.

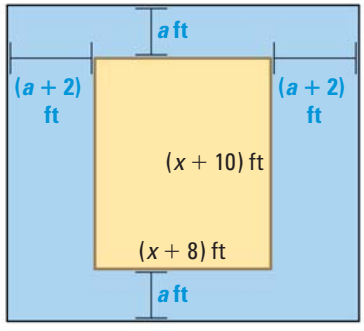
c. Interpreting a Graph Based on your graph, do you expect the keys to reach your friend? *Explain* your answer.

d. Using an Equation Find the value of x when $y = 20$. (You may need to factor out a -1 in order to factor the trinomial.) What do you notice? *Explain* how the x-value justifies your answer from part (c).

65. **CHALLENGE** A rectangular stage is positioned in the center of a rectangular room, as shown. The area of the stage is 120 square feet.

a. Use the dimensions given in the diagram to find the length and width of the stage.

b. The combined area of the stage and the surrounding floor is 360 square feet. Find the length and width of the room.



MIXED REVIEW

Solve the equation.

66. $x + 12 = 4$ *(p. 134)*

67. $5y - 2 = 13$ *(p. 141)*

68. $6n + 4 = -14$ *(p. 141)*

69. $3a - 5a + 12 = -6$ *(p. 148)*

70. $3 - 2(w + 7) = -1$ *(p. 148)*

71. $-6 + 2(d - 9) = 8d$ *(p. 154)*

72. $(x - 8)(x + 3) = 0$ *(p. 575)*

73. $(3t + 5)(t + 2) = 0$ *(p. 575)*

Find the product.

74. $(3x + 7)(x - 5)$ *(p. 562)*

75. $(3a - 4)(2a - 9)$ *(p. 562)*

76. $(c + 2)(c^2 + c - 4)$ *(p. 562)*

77. $(7 + 3y)(7 - 5y)$ *(p. 562)*

78. $(2k - 8)(2k + 8)$ *(p. 569)*

79. $(14 - 2n)^2$ *(p. 569)*

80. $(5x + 16y)^2$ *(p. 569)*

81. $(3x - 6y)(3x + 6y)$ *(p. 569)*

PREVIEW
Prepare for Lesson 9.6 in Exs. 74–81.

PROBLEM SOLVING WORKSHOP
LESSON 9.5

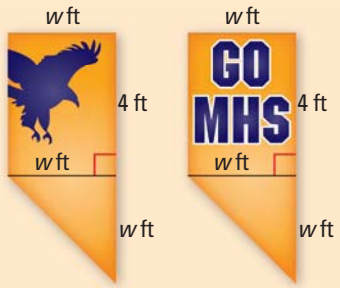
Using ALTERNATIVE METHODS

Another Way to Solve Example 5, page 585

MULTIPLE REPRESENTATIONS In Example 5 on page 585, you saw how to solve the problem about a school banner by solving an equation. You can also solve the problem using a table or a graph.

PROBLEM

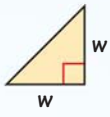
BANNER DIMENSIONS You are making banners to hang during school spirit week. Each banner requires 16.5 square feet of felt and will be cut as shown. Find the width of one banner.



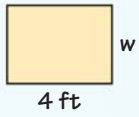
METHOD 1

Using a Table Consider the separate geometric figures that form one banner and find their areas in terms of w. Then find the total area of the banner for different values of w until you find a value that gives a total area of 16.5 square feet. Use a table to organize your work.

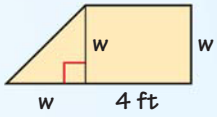
STEP 1 **Write** equations for the area of the pieces and the total area.



$A = \frac{1}{2}w^2$



$A = 4w$



$A = \frac{1}{2}w^2 + 4w$

STEP 2 **Organize** your work in a table.

w	Triangle's area $\left(\frac{1}{2}w^2\right)$	Rectangle's area $(4w)$	Total area $\left(\frac{1}{2}w^2 + 4w\right)$	
1	0.5	4	4.5	← 4.5 < 16.5, so try a greater value of w.
2	2	8	10	← 10 < 16.5, so try a greater value of w.
3	4.5	12	16.5	← Correct area

▶ The width of the banner is 3 feet.

METHOD 2

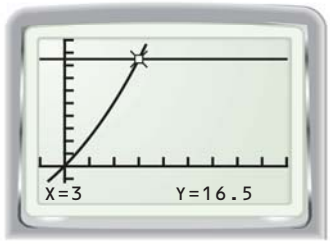
Using a Graph Another approach is to use a graph.

STEP 1 **Write** an equation for the area of the banner. The area of the banner can be thought of as the area of a triangle plus the area of a rectangle.

$$\text{Area of banner} = \text{Area of triangle} + \text{Area of rectangle}$$

$$A = \frac{1}{2}w^2 + 4w$$

STEP 2 **Graph** the equation for the area of the banner using a graphing calculator. Graph $y_1 = 0.5x^2 + 4x$. Because you are looking for the value of x that gives an area of 16.5 square feet, you should display the graph of $y_2 = 16.5$ in the same viewing window.

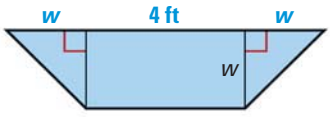


STEP 3 **Find** the intersection of the graphs by using the *intersect* feature on your calculator. The graphs intersect at (3, 16.5).

▶ The width of the banner is 3 feet.

PRACTICE

1. **COUNTER DESIGN** A contractor is building a counter in a kitchen using the diagram shown. The countertop will have an area of 12 square feet. How wide should it be? Solve this problem using two different methods.

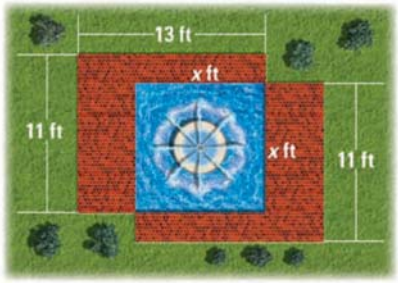


2. **ERROR ANALYSIS** *Describe* and correct the error in using an equation to solve the problem in Exercise 1.

$$12 = 4w + \frac{1}{2}w^2 + \frac{1}{2}w^2$$
$$0 = w^2 + 4w - 12$$
$$0 = (w + 2)(w - 6)$$
$$w + 2 = 0 \quad \text{or} \quad w - 6 = 0$$
$$w = -2 \quad \text{or} \quad w = 6$$

The width is 6 feet.

3. **FOUNTAIN DESIGN** A square fountain in a city plaza is surrounded by brick patios as shown. The combined area of the fountain and brick patios is 205 square feet. What is the side length of the fountain? Solve this problem using two different methods.



4. **WHAT IF?** You want to make a larger banner using the same pattern shown in the problem on page 585. The new banner will have an area of 24 square feet. Find the width of the new banner. *Describe* the method you used to find your answer.

9.6 More Factorization with Algebra Tiles

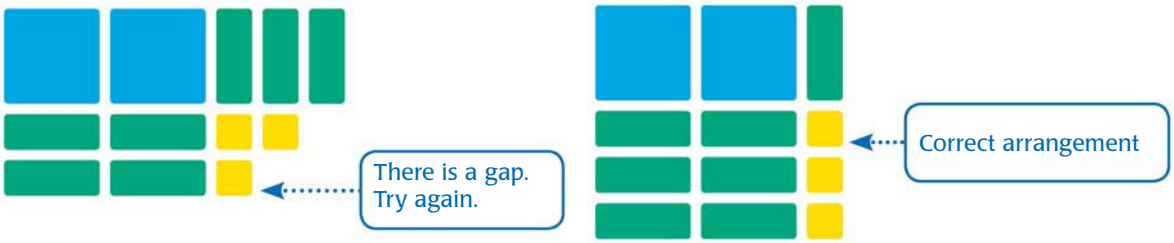
MATERIALS • algebra tiles

QUESTION How can you factor a trinomial using algebra tiles?

EXPLORE Factor the trinomial $2x^2 + 7x + 3$

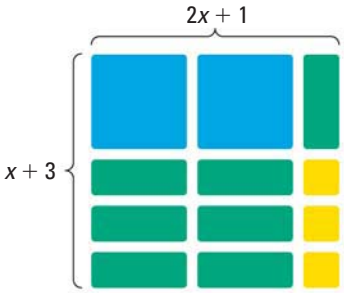
STEP 1 *Make a rectangle*

Model the trinomial with algebra tiles. Arrange all of the tiles to form a rectangle. You may have to try a few arrangements to make the rectangle. There can be no gaps or leftover tiles.



STEP 2 *Find the side lengths*

The side lengths of the rectangle represent the polynomials $x + 3$ and $2x + 1$. So $2x^2 + 7x + 3 = (x + 3)(2x + 1)$.



DRAW CONCLUSIONS Use your observations to complete these exercises

1. Use multiplication to show that $x + 3$ and $2x + 1$ are factors of the polynomial $2x^2 + 7x + 3$.

Use algebra tiles to factor the trinomial. Include a drawing of your model.

2. $2x^2 + 5x + 3$
3. $3x^2 + 5x + 2$
4. $4x^2 + 9x + 2$
5. $3x^2 + 13x + 4$
6. $4x^2 + 11x + 6$
7. $4x^2 + 8x + 3$
8. **REASONING** Factor the trinomial $2x^2 + 11x + 5$ into two binomials. How is the leading coefficient of the trinomial related to the leading coefficients of its binomial factors?

9.6 Factor $ax^2 + bx + c$

Before You factored trinomials of the form $x^2 + bx + c$.

Now You will factor trinomials of the form $ax^2 + bx + c$.

Why? So you can find the dimensions of a building, as in Ex. 61.

Key Vocabulary
- **trinomial,** *p. 555*

When factoring a trinomial of the form $ax^2 + bx + c$, first consider the signs of b and c, as in Lesson 9.5. This approach works when a is positive.

EXAMPLE 1 Factor when b is negative and c is positive

Factor $2x^2 - 7x + 3$.

Solution

REVIEW FACTORING
For help with determining the signs of the factors of a trinomial, see p. 584.

Because b is negative and c is positive, both factors of c must be negative. Make a table to organize your work.

You must consider the order of the factors of 3, because the x-terms of the possible factorizations are different.

Factors of 2	Factors of 3	Possible factorization	Middle term when multiplied	
1, 2	−1, −3	$(x - 1)(2x - 3)$	$-3x - 2x = -5x$	✗
1, 2	−3, −1	$(x - 3)(2x - 1)$	$-x - 6x = -7x$	← Correct

▶ $2x^2 - 7x + 3 = (x - 3)(2x - 1)$

EXAMPLE 2 Factor when b is positive and c is negative

Factor $3n^2 + 14n - 5$.

Solution

Because b is positive and c is negative, the factors of c have different signs.

Factors of 3	Factors of −5	Possible factorization	Middle term when multiplied	
1, 3	1, −5	$(n + 1)(3n - 5)$	$-5n + 3n = -2n$	✗
1, 3	−1, 5	$(n - 1)(3n + 5)$	$5n - 3n = 2n$	✗
1, 3	5, −1	$(n + 5)(3n - 1)$	$-n + 15n = 14n$	← Correct
1, 3	−5, 1	$(n - 5)(3n + 1)$	$n - 15n = -14n$	✗

▶ $3n^2 + 14n - 5 = (n + 5)(3n - 1)$

GUIDED PRACTICE for Examples 1 and 2

Factor the trinomial.

1. $3t^2 + 8t + 4$ **2.** $4s^2 - 9s + 5$ **3.** $2h^2 + 13h - 7$

FACTORING WHEN *a* IS NEGATIVE To factor a trinomial of the form $ax^2 + bx + c$ when a is negative, first factor -1 from each term of the trinomial. Then factor the resulting trinomial as in the previous examples.

EXAMPLE 3 Factor when *a* is negative

Factor $-4x^2 + 12x + 7$.

Solution

STEP 1 **Factor** -1 from each term of the trinomial.

$$-4x^2 + 12x + 7 = -(4x^2 - 12x - 7)$$

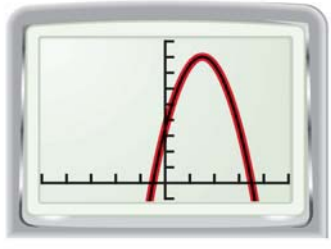
STEP 2 **Factor** the trinomial $4x^2 - 12x - 7$. Because b and c are both negative, the factors of c must have different signs. As in the previous examples, use a table to organize information about the factors of a and c.

Factors of 4	Factors of −7	Possible factorization	Middle term when multiplied	
1, 4	1, −7	$(x + 1)(4x - 7)$	$-7x + 4x = -3x$	✗
1, 4	7, −1	$(x + 7)(4x - 1)$	$-x + 28x = 27x$	✗
1, 4	−1, 7	$(x - 1)(4x + 7)$	$7x - 4x = 3x$	✗
1, 4	−7, 1	$(x - 7)(4x + 1)$	$x - 28x = -27x$	✗
2, 2	1, −7	$(2x + 1)(2x - 7)$	$-14x + 2x = -12x$	← Correct
2, 2	−1, 7	$(2x - 1)(2x + 7)$	$14x - 2x = 12x$	✗

AVOID ERRORS Remember to include the -1 that you factored out in Step 1.

▶ $-4x^2 + 12x + 7 = -(2x + 1)(2x - 7)$

CHECK You can check your factorization using a graphing calculator. Graph $y_1 = -4x^2 + 12x + 7$ and $y_2 = -(2x + 1)(2x - 7)$. Because the graphs coincide, you know that your factorization is correct.



GUIDED PRACTICE for Example 3

Factor the trinomial.

4. $-2y^2 - 5y - 3$ **5.** $-5m^2 + 6m - 1$ **6.** $-3x^2 - x + 2$

FINDING A COMMON FACTOR In Lesson 9.4, you learned to factor out the greatest common monomial factor from the terms of a polynomial. Sometimes you may need to do this before finding two binomial factors of a trinomial.

EXAMPLE 4 Write and solve a polynomial equation

DISCUS An athlete throws a discus from an initial height of 6 feet and with an initial vertical velocity of 46 feet per second.

a. Write an equation that gives the height (in feet) of the discus as a function of the time (in seconds) since it left the athlete's hand.

b. After how many seconds does the discus hit the ground?

Solution

a. Use the vertical motion model to write an equation for the height h (in feet) of the discus. In this case, $v = 46$ and $s = 6$.

$h = -16t^2 + vt + s$ **Vertical motion model**

$h = -16t^2 + 46t + 6$ **Substitute 46 for *v* and 6 for *s*.**

USE VERTICAL MOTION MODEL
For help with using the vertical motion model, see p. 575.

b. To find the number of seconds that pass before the discus lands, find the value of t for which the height of the discus is 0. Substitute 0 for h and solve the equation for t.

$0 = -16t^2 + 46t + 6$ **Substitute 0 for *h*.**

$0 = -2(8t^2 - 23t - 3)$ **Factor out −2.**

$0 = -2(8t + 1)(t - 3)$ **Factor the trinomial. Find factors of 8 and −3 that produce a middle term with a coefficient of −23.**

$8t + 1 = 0$ *or* $t - 3 = 0$ **Zero-product property**

$t = -\frac{1}{8}$ *or* $t = 3$ **Solve for *t*.**

The solutions of the equation are $-\frac{1}{8}$ and 3. A negative solution does not make sense in this situation, so disregard $-\frac{1}{8}$.

▶ The discus hits the ground after 3 seconds.

GUIDED PRACTICE for Example 4

7. WHAT IF? In Example 4, suppose another athlete throws the discus with an initial vertical velocity of 38 feet per second and releases it from a height of 5 feet. After how many seconds does the discus hit the ground?

8. SHOT PUT In a shot put event, an athlete throws the shot put from an initial height of 6 feet and with an initial vertical velocity of 29 feet per second. After how many seconds does the shot put hit the ground?

EXAMPLE 5 Standardized Test Practice

A rectangle's length is 13 meters more than 3 times its width. The area is 10 square meters. What is the width?

Ⓐ $\frac{2}{3}$ m Ⓑ 3 m Ⓒ 5 m Ⓓ 10 m

$w(3w + 13) = 10$ **Write an equation to model area.**

$3w^2 + 13w - 10 = 0$ **Simplify and subtract 10 from each side.**

$(w + 5)(3w - 2) = 0$ **Factor left side.**

$w + 5 = 0$ *or* $3w - 2 = 0$ **Zero-product property**

$w = -5$ *or* $w = \frac{2}{3}$ **Solve for *w*.**

Reject the negative width.

▶ The correct answer is A. Ⓐ Ⓑ Ⓒ Ⓓ

GUIDED PRACTICE for Example 5

9. A rectangle's length is 1 inch more than twice its width. The area is 6 square inches. What is the width?

Ⓐ $\frac{1}{2}$ in. Ⓑ $\frac{3}{2}$ in. Ⓒ 2 in. Ⓓ $\frac{5}{2}$ in.

9.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 25, and 61

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 3, 22, 41, 51, and 61

◆ = **MULTIPLE REPRESENTATIONS** Ex. 62

SKILL PRACTICE

1. VOCABULARY What is another word for the solutions of $x^2 + 2x + 1 = 0$?

2. ★ WRITING *Explain* how you can use a graph to check a factorization.

3. ★ WRITING *Compare* factoring $6x^2 - x - 2$ with factoring $x^2 - x - 2$.

EXAMPLES 1, 2, and 3 on pp. 593–594 for Exs. 4–22

FACTORING TRINOMIALS **Factor the trinomial.**

4. $-x^2 + x + 20$ **5.** $-y^2 + 2y + 8$ **6.** $-a^2 + 12a - 27$

7. $5w^2 - 6w + 1$ **8.** $-3p^2 - 10p - 3$ **9.** $6s^2 - s - 5$

10. $2t^2 + 5t - 63$ **11.** $2c^2 - 7c + 3$ **12.** $3n^2 - 17n + 10$

13. $-2h^2 + 5h + 3$ **14.** $-6k^2 - 13k - 6$ **15.** $10x^2 - 3x - 27$

16. $4m^2 + 9m + 5$ **17.** $3z^2 + z - 14$ **18.** $4a^2 + 9a - 9$

19. $4n^2 + 16n + 15$ **20.** $-5b^2 + 7b - 2$ **21.** $6y^2 - 5y - 4$

22. ★ **MULTIPLE CHOICE** What is the correct factorization of $8x^2 - 10x + 3$?

Ⓐ $(2x - 3)(4x - 1)$ Ⓑ $(2x - 1)(4x - 3)$

Ⓒ $(4x + 1)(2x - 3)$ Ⓓ $(8x - 3)(x - 1)$

EXAMPLES 4 and 5 on pp. 595–596 for Exs. 23–39

SOLVING EQUATIONS **Solve the equation.**

23. $2x^2 - 3x - 35 = 0$ **24.** $3w^2 + 22w + 7 = 0$ **25.** $4s^2 + 11s - 3 = 0$

26. $7a^2 + 2a = 5$ **27.** $8t^2 - 2t = 3$ **28.** $6m^2 - 5m = 14$

29. $b(20b - 3) - 2 = 0$ **30.** $4(3y^2 - 7y + 4) = 1$ **31.** $p(3p + 14) = 5$

32. $4n^2 - 2n - 90 = 0$ **33.** $10c^2 - 14c + 4 = 0$ **34.** $-16k^2 + 8k + 24 = 0$

35. $6r^2 - 15r = 99$ **36.** $56z^2 + 2 = 22z$ **37.** $30x^2 + 25x = 20$

ERROR ANALYSIS ***Describe*** **and correct the error in solving the equation.**

38.

$5x^2 + x = 4$ ✗

$x(5x + 1) = 4$

$x = 4$ *or* $5x + 1 = 4$

$x = 4$ *or* $x = \frac{3}{5}$

39.

$12x^2 + 5x - 2 = 0$ ✗

$(3x - 1)(4x + 2) = 0$

$3x - 1 = 0$ *or* $4x + 2 = 0$

$x = \frac{1}{3}$ *or* $x = -\frac{1}{2}$

40. **GEOMETRY** The length of a rectangle is 7 inches more than 5 times its width. The area of the rectangle is 6 square inches. What is the width?

41. ★ **SHORT RESPONSE** The length of a rectangle is 1 inch more than 4 times its width. The area of the rectangle is 3 square inches. What is the perimeter of the rectangle? *Explain* how you found your answer.

FINDING ZEROS OF FUNCTIONS **Find the zeros of the polynomial function.**

42. $g(x) = 2x^2 + x - 1$ **43.** $f(x) = -x^2 + 12x - 35$ **44.** $h(x) = -3x^2 + 2x + 5$

45. $f(x) = 3x^2 + x - 14$ **46.** $g(x) = 8x^2 - 6x - 14$ **47.** $f(x) = 12x^2 - 24x - 63$

SOLVING EQUATIONS **Multiply each side of the equation by an appropriate power of 10 to obtain integer coefficients. Then solve the equation.**

48. $0.3x^2 - 0.7x - 4.0 = 0$ **49.** $0.8x^2 - 1.8x - 0.5 = 0$ **50.** $0.4x^2 - 0.4x = 9.9$

51. ★ **MULTIPLE CHOICE** What are the solutions of the equation $0.4x^2 - 1.1x = 2$?

Ⓐ -12.5 and 40 Ⓑ -4 and 1.25 Ⓒ -1.25 and 4 Ⓓ -0.125 and 0.4

WRITING EQUATIONS **Write a polynomial equation that has the given solutions. The equation must have integer coefficients.** ***Explain*** **your reasoning.**

52. -3 and 2 **53.** $-\frac{1}{2}$ and 5 **54.** $-\frac{3}{4}$ and $-\frac{1}{3}$

CHALLENGE **Factor the trinomial.**

55. $2x^2 - 11xy + 5y^2$ **56.** $3x^2 + 2xy - 8y^2$ **57.** $6x^3 - 10x^2y - 56xy^2$

PROBLEM SOLVING

EXAMPLE 4
on p. 595
for Exs. 58, 60

58. DIVING You dive from a platform when your center of gravity is 32 feet above the surface of a swimming pool. Your initial vertical velocity leaving the platform is 28 feet per second. After how many seconds does your center of gravity enter the water?

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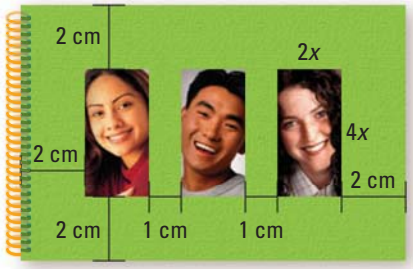
EXAMPLE 5
on p. 596
for Exs. 59, 61

59. SCRAPBOOK DESIGN You plan to make a scrapbook. On the cover, you want to show three pictures with space between them, as shown. Each of the pictures is twice as long as it is wide.



a. Write a polynomial that represents the area of the scrapbook cover.

b. The area of the cover will be 96 square centimeters. Find the length and width of the pictures you will use.

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60. ★ SHORT RESPONSE You throw a ball into the air with an initial vertical velocity of 31 feet per second. The ball leaves your hand when it is 6 feet above the ground. You catch the ball when it reaches a height of 4 feet. After how many seconds do you catch the ball? *Explain* how you can use the solutions of an equation to find your answer.

61. PARTHENON The Parthenon in Athens, Greece, is an ancient structure that has a rectangular base. The length of the Parthenon's base is 8 meters more than twice its width. The area of the base is about 2170 square meters. Find the length and width of the Parthenon's base.

62. MULTIPLE REPRESENTATIONS An African cat called a serval leaps from the ground in an attempt to catch a bird. The serval's initial vertical velocity is 24 feet per second.

a. Writing an Equation Write an equation that gives the serval's height (in feet) as a function of the time (in seconds) since it left the ground.

b. Making a Table Use the equation from part (a) to make a table that shows the height of the serval for $t = 0$, 0.3, 0.6, 0.9, 1.2, and 1.5 seconds.

c. Drawing a Graph Plot the ordered pairs in the table as points in a coordinate plane. Connect the points with a smooth curve. After how many seconds does the serval reach a height of 9 feet? *Justify* your answer using the equation from part (a).

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○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE ◆ = MULTIPLE REPRESENTATIONS

63. CHALLENGE A bush cricket jumps from the ground into the air with an initial vertical velocity of 4 feet per second.

a. Write an equation that gives the cricket's height (in feet) as a function of the time (in seconds) since it left the ground.

b. After how many seconds is the cricket 3 inches off the ground?

c. Does the cricket jump higher than 3 inches? *Explain* your reasoning using your answer from part (b).

MIXED REVIEW

Check whether the given number is a solution of the equation or inequality.

64. $b - 9 = 18; 3$ *(p. 21)*

65. $8 - 3h = 2; 2$ *(p. 21)*

66. $\frac{28 - 2x}{x} < 5; 4$ *(p. 21)*

67. $6t + 18 = 0; -3$ *(p. 21)*

68. $6c = 3c; 2$ *(p. 21)*

69. $|x + 3| = 2; -5$ *(p. 64)*

70. $|y - 2| + 6 = 5; 1$ *(p. 64)*

71. $|3n - 11| < 1; 2$ *(p. 64)*

72. $4|3a - 8| > 2; 3$ *(p. 64)*

PREVIEW
Prepare for Lesson 9.7 in Exs. 73–81.

Find the product. *(p. 569)*

73. $(a - 9)^2$

74. $(k + 12)^2$

75. $(3x - 2)^2$

76. $(m + 4)(m - 4)$

77. $(2c + 1)(2c - 1)$

78. $(5n - 3)(5n + 3)$

79. $(8 - 3y)^2$

80. $(2s - 5t)^2$

81. $(x + 2y)(x - 2y)$

QUIZ *for Lessons 9.4–9.6*

Factor out the greatest common monomial factor. *(p. 575)*

1. $16a^2 - 40b$

2. $9xy^2 + 6x^2y$

3. $4n^4 - 22n^3 - 8n^2$

4. $3x^2 + 6xy - 3y^2$

5. $12abc^2 - 6a^2c$

6. $-36s^3 + 18s^2 - 54s$

Factor the trinomial.

7. $r^2 + 15r + 56$ *(p. 583)*

8. $s^2 - 6s + 5$ *(p. 583)*

9. $w^2 + 6w - 40$ *(p. 583)*

10. $-a^2 + 9a + 22$ *(p. 593)*

11. $2x^2 - 9x + 4$ *(p. 593)*

12. $5m^2 + m - 6$ *(p. 593)*

13. $6h^2 - 19h + 3$ *(p. 593)*

14. $-7y^2 - 23y - 6$ *(p. 593)*

15. $18c^2 + 12c - 6$ *(p. 593)*

Solve the equation.

16. $(4p - 7)(p + 5) = 0$ *(p. 575)*

17. $-8u^2 + 28u = 0$ *(p. 575)*

18. $51x^2 = -17x$ *(p. 575)*

19. $b^2 - 11b = -24$ *(p. 583)*

20. $m^2 + 12m = -35$ *(p. 583)*

21. $q^2 + 19 = -20q$ *(p. 583)*

22. $3t^2 - 11t + 10 = 0$ *(p. 593)*

23. $4y^2 + 31y = 8$ *(p. 593)*

24. $14s^2 + 12s = 2$ *(p. 593)*

25. BASEBALL A baseball player hits a baseball into the air with an initial vertical velocity of 72 feet per second. The player hits the ball from a height of 3 feet. *(p. 593)*

a. Write an equation that gives the baseball's height as a function of the time (in seconds) after it is hit.

b. After how many seconds is the baseball 84 feet above the ground?

9.7 Factor Special Products

Before You factored polynomials of the form $ax^2 + bx + c$.

Now You will factor special products.

Why? So you can use a scientific model, as in Ex. 48.

Key Vocabulary
- **perfect square trinomial**

You can use the special product patterns you studied in Lesson 9.3 to factor polynomials, such as the difference of two squares.

KEY CONCEPT *For Your Notebook*

Difference of Two Squares Pattern

Algebra	Example
$a^2 - b^2 = (a + b)(a - b)$	$4x^2 - 9 = (2x)^2 - 3^2 = (2x + 3)(2x - 3)$

EXAMPLE 1 Factor the difference of two squares

Factor the polynomial.

a. $y^2 - 16 = y^2 - 4^2$ — Write as $a^2 - b^2$.

$= (y + 4)(y - 4)$ — Difference of two squares pattern

b. $25m^2 - 36 = (5m)^2 - 6^2$ — Write as $a^2 - b^2$.

$= (5m + 6)(5m - 6)$ — Difference of two squares pattern

c. $x^2 - 49y^2 = x^2 - (7y)^2$ — Write as $a^2 - b^2$.

$= (x + 7y)(x - 7y)$ — Difference of two squares pattern

EXAMPLE 2 Factor the difference of two squares

Factor the polynomial $8 - 18n^2$.

$8 - 18n^2 = 2(4 - 9n^2)$ — Factor out common factor.

$= 2[2^2 - (3n)^2]$ — Write $4 - 9n^2$ as $a^2 - b^2$.

$= 2(2 + 3n)(2 - 3n)$ — Difference of two squares pattern

✓ GUIDED PRACTICE for Examples 1 and 2

1. Factor the polynomial $4y^2 - 64$.

PERFECT SQUARE TRINOMIALS The pattern for finding the square of a binomial gives you the pattern for factoring trinomials of the form $a^2 + 2ab + b^2$ and $a^2 - 2ab + b^2$. These are called **perfect square trinomials**.

KEY CONCEPT *For Your Notebook*

Perfect Square Trinomial Pattern

Algebra	Example
$a^2 + 2ab + b^2 = (a + b)^2$	$x^2 + 6x + 9 = x^2 + 2(x \cdot 3) + 3^2 = (x + 3)^2$
$a^2 - 2ab + b^2 = (a - b)^2$	$x^2 - 10x + 25 = x^2 - 2(x \cdot 5) + 5^2 = (x - 5)^2$

EXAMPLE 3 Factor perfect square trinomials

Factor the polynomial.

a. $n^2 - 12n + 36 = n^2 - 2(n \cdot 6) + 6^2$ — Write as $a^2 - 2ab + b^2$.

$= (n - 6)^2$ — Perfect square trinomial pattern

b. $9x^2 - 12x + 4 = (3x)^2 - 2(3x \cdot 2) + 2^2$ — Write as $a^2 - 2ab + b^2$.

$= (3x - 2)^2$ — Perfect square trinomial pattern

c. $4s^2 + 4st + t^2 = (2s)^2 + 2(2s \cdot t) + t^2$ — Write as $a^2 + 2ab + b^2$.

$= (2s + t)^2$ — Perfect square trinomial pattern

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EXAMPLE 4 Factor a perfect square trinomial

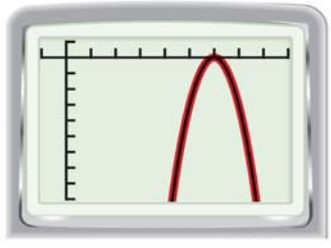
Factor the polynomial $-3y^2 + 36y - 108$.

$-3y^2 + 36y - 108 = -3(y^2 - 12y + 36)$ — Factor out -3.

$= -3[y^2 - 2(y \cdot 6) + 6^2]$ — Write $y^2 - 12y + 36$ as $a^2 - 2ab + b^2$.

$= -3(y - 6)^2$ — Perfect square trinomial pattern

CHECK Check your factorization using a graphing calculator. Graph $y_1 = -3x^2 + 36x - 108$ and $y_2 = -3(x - 6)^2$. Because the graphs coincide, you know that your factorization is correct.



GUIDED PRACTICE for Examples 3 and 4

Factor the polynomial.

2. $h^2 + 4h + 4$ **3.** $2y^2 - 20y + 50$ **4.** $3x^2 + 6xy + 3y^2$

EXAMPLE 5 Solve a polynomial equation

Solve the equation $x^2 + \frac{2}{3}x + \frac{1}{9} = 0$.

$$x^2 + \frac{2}{3}x + \frac{1}{9} = 0$$ Write original equation.

$$9x^2 + 6x + 1 = 0$$ Multiply each side by 9.

$$(3x)^2 + 2(3x \cdot 1) + (1)^2 = 0$$ Write left side as $a^2 + 2ab + b^2$.

$$(3x + 1)^2 = 0$$ Perfect square trinomial pattern

$$3x + 1 = 0$$ Zero-product property

$$x = -\frac{1}{3}$$ Solve for x.

FIND SOLUTIONS
This equation has two identical solutions, because it has two identical factors.

▶ The solution of the equation is $-\frac{1}{3}$.

EXAMPLE 6 Solve a vertical motion problem

FALLING OBJECT A window washer drops a wet sponge from a height of 64 feet. After how many seconds does the sponge land on the ground?

Solution

Use the vertical motion model to write an equation for the height h (in feet) of the sponge as a function of the time t (in seconds) after it is dropped.

The sponge was dropped, so it has no initial vertical velocity. Find the value of t for which the height is 0.

$$h = -16t^2 + vt + s$$ Vertical motion model

$$0 = -16t^2 + (0)t + 64$$ Substitute 0 for h, 0 for v, and 64 for s.

$$0 = -16(t^2 - 4)$$ Factor out -16.

$$0 = -16(t - 2)(t + 2)$$ Difference of two squares pattern

$$t - 2 = 0 \quad or \quad t + 2 = 0$$ Zero-product property

$$t = 2 \quad or \quad t = -2$$ Solve for t.

Disregard the negative solution of the equation.

▶ The sponge lands on the ground 2 seconds after it is dropped.

GUIDED PRACTICE for Examples 5 and 6

Solve the equation.

5. $a^2 + 6a + 9 = 0$ **6.** $w^2 - 14w + 49 = 0$ **7.** $n^2 - 81 = 0$

8. WHAT IF? In Example 6, suppose the sponge is dropped from a height of 16 feet. After how many seconds does it land on the ground?

9.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11 and 49

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 23, 24, 49, and 50

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The polynomial $9n^2 + 6n + 1$ is called a(n) _?_ trinomial.

2. ★ **WRITING** *Explain* how to factor the difference of two squares.

EXAMPLES 1 and 2 on p. 600 for Exs. 3–8

DIFFERENCE OF TWO SQUARES **Factor the polynomial.**

3. $x^2 - 25$
4. $n^2 - 64$
5. $81c^2 - 4$
6. $49 - 121p^2$
7. $-3m^2 + 48n^2$
8. $225x^2 - 144y^2$

EXAMPLES 3 and 4 on p. 601 for Exs. 9–14

PERFECT SQUARE TRINOMIALS **Factor the polynomial.**

9. $x^2 - 4x + 4$
10. $y^2 - 10y + 25$
11. $49a^2 + 14a + 1$
12. $9t^2 - 12t + 4$
13. $m^2 + m + \frac{1}{4}$
14. $2x^2 + 12xy + 18y^2$

EXAMPLES 1, 2, 3, and 4 on pp. 600–601 for Exs. 15–24

FACTORING POLYNOMIALS **Factor the polynomial.**

15. $4c^2 - 400$
16. $4f^2 - 36f + 81$
17. $-9r^2 + 4s^2$
18. $z^2 + 12z + 36$
19. $72 - 32y^2$
20. $45r^2 - 120rs + 80s^2$

ERROR ANALYSIS ***Describe*** **and correct the error in factoring.**

21.
$$36x^2 - 81 = 9(4x^2 - 9)$$
$$= 9((2x)^2 - 3^2)$$
$$= 9(2x - 3)^2$$

22.
$$y^2 - 6y + 9 = y^2 - 2(y \cdot 3) + 3^2$$
$$= (y - 3)(y + 3)$$

23. ★ **MULTIPLE CHOICE** Which is the correct factorization of $-45x^2 + 20y^2$?

(A) $-5(3x + 2y)^2$ (B) $5(3x - 2y)^2$

(C) $-5(3x + 2y)(3x - 2y)$ (D) $5(3x + 2y)(3x - 2y)$

24. ★ **MULTIPLE CHOICE** Which is the correct factorization of $16m^2 - 8mn + n^2$?

(A) $(4m - n)^2$ (B) $(4m + n)^2$

(C) $(8m - n)^2$ (D) $(4m - n)(4m + n)$

EXAMPLE 5 on p. 602 for Exs. 25–39

SOLVING EQUATIONS **Solve the equation.**

25. $x^2 + 8x + 16 = 0$
26. $16a^2 - 8a + 1 = 0$
27. $4w^2 - 36 = 0$
28. $32 - 18m^2 = 0$
29. $27c^2 + 108c + 108 = 0$
30. $-2h^2 - 28h - 98 = 0$
31. $6p^2 = 864$
32. $-3t^2 = -108$
33. $8k^2 = 98$
34. $-\frac{4}{3}x + \frac{4}{9} = -x^2$
35. $y^2 - \frac{5}{3}y = -\frac{25}{36}$
36. $\frac{2}{9} = 8n^2$
37. $-9c^2 = -16$
38. $-20s - 3 = 25s^2 + 1$
39. $y^4 - 2y^3 + y^2 = 0$

CHALLENGE **Determine the value(s) of k for which the expression is a perfect square trinomial.**

40. $x^2 + kx + 36$ **41.** $4x^2 + kx + 9$ **42.** $16x^2 + kx + 4$

43. $25x^2 + 10x + k$ **44.** $49x^2 - 84x + k$ **45.** $4x^2 - 48x + k$

PROBLEM SOLVING

EXAMPLE 6 on p. 602 for Exs. 46–48

46. FALLING BRUSH While standing on a ladder, you drop a paintbrush from a height of 25 feet. After how many seconds does the paintbrush land on the ground?

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47. FALLING OBJECT A hickory nut falls from a branch that is 100 feet above the ground. After how many seconds does the hickory nut land on the ground?

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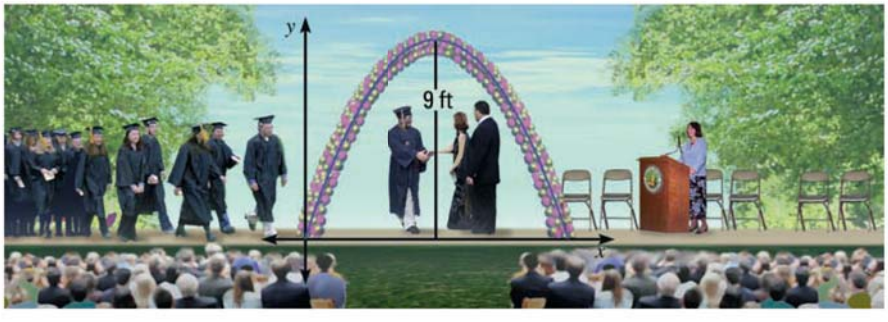
48. GRASSHOPPER A grasshopper jumps straight up from the ground with an initial vertical velocity of 8 feet per second.

a. Write an equation that gives the height (in feet) of the grasshopper as a function of the time (in seconds) since it leaves the ground.

b. After how many seconds is the grasshopper 1 foot off the ground?

49. ★ **SHORT RESPONSE** A ball is thrown up into the air from a height of 5 feet with an initial vertical velocity of 56 feet per second. How many times does the ball reach a height of 54 feet? *Explain* your answer.

50. ★ **EXTENDED RESPONSE** An arch of balloons decorates the stage at a high school graduation. The balloons are tied to a frame. The shape of the frame can be modeled by the graph of the equation $y = -\frac{1}{4}x^2 + 3x$ where x and y are measured in feet.



a. Make a table of values that shows the height of the balloon arch for $x = 0, 2, 5, 8,$ and 11 feet.

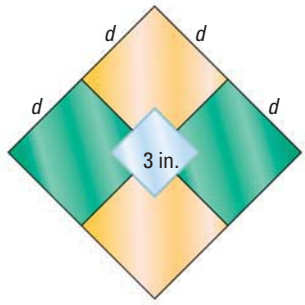
b. For what additional values of x does the equation make sense? *Explain*.

c. At approximately what distance from the left end does the arch reach a height of 9 feet? Check your answer algebraically.

○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

51. FRAMING A square mirror is framed with stained glass as shown. Each corner of the frame began as a square with a side length of d inches before it was cut to fit the mirror. The mirror has a side length of 3 inches. The area of the stained glass frame is 91 square inches.



a. Write a polynomial that represents the area of the stained glass frame.

b. What is the side length of the frame?

52. CHALLENGE You have 120 folding chairs to set up in a park for an outdoor play. You want each row to have an odd number of chairs. You also want each row after the first to have 2 more chairs than the row in front of it. The first row will have 15 chairs.

a. Copy and complete the table below.

n	nth odd integer	Sum of first n odd integers	Sum as a power
1	1	1	1^2
2	3	$1 + 3 = 4$	2^2
3	5	$1 + 3 + 5 = 9$	?
4	7	?	?
5	9	?	?

b. *Describe* the relationship between n and the sum of the first n odd integers. Then find the sum of the first 10 odd integers.

c. *Explain* how to find the sum of the odd integers from 11 to 21.

d. How many rows of chairs will you need for the outdoor play? *Explain* your thinking.

MIXED REVIEW

PREVIEW
Prepare for Lesson 9.8 in Exs. 53–61.

Solve the equation.

53. $a + 6 = 3$ *(p. 134)*

54. $5y - 2 = -32$ *(p. 141)*

55. $8m + 4 = 20$ *(p. 141)*

56. $5b - 3b + 6 = 4$ *(p. 148)*

57. $(x - 9)(x + 1) = 0$ *(p. 575)*

58. $x^2 + 17x = -66$ *(p. 583)*

59. $x^2 = -12x + 45$ *(p. 583)*

60. $2y^2 + y = 15$ *(p. 593)*

61. $22z - 35 = 3z^2$ *(p. 593)*

Graph the linear equation.

62. $y = -6$ *(p. 215)*

63. $x = 14$ *(p. 215)*

64. $2x + y = 8$ *(p. 225)*

65. $-4x + 5y = -20$ *(p. 225)*

66. $0.6x + 0.2y = 3.6$ *(p. 225)*

67. $y = -\frac{3}{2}x - 9$ *(p. 225)*

68. $y = \frac{5}{2}x$ *(p. 244)*

69. $y = -12x + 3$ *(p. 244)*

70. $y = \frac{4}{3}x + 2$ *(p. 244)*

Find the product.

71. $(2a - 3)(5a - 2)$ *(p. 562)*

72. $(2x^2 + x + 3)(x - 1)$ *(p. 562)*

73. $(c + 3)(c + 5)$ *(p. 562)*

74. $(3x - 4)(2x - 7)$ *(p. 562)*

75. $(2k - 11)(2k + 11)$ *(p. 569)*

76. $(y - 7)^2$ *(p. 569)*

9.8 Factor Polynomials Completely

Before You factored polynomials.

Now You will factor polynomials completely.

Why? So you can model the height of a projectile, as in Ex. 71.

Key Vocabulary
- **factor by grouping**
- **factor completely**

You have used the distributive property to factor a greatest common monomial from a polynomial. Sometimes, you can factor out a common binomial.

EXAMPLE 1 Factor out a common binomial

Factor the expression.

a. $2x(x + 4) - 3(x + 4)$ **b.** $3y^2(y - 2) + 5(2 - y)$

Solution

a. $2x(x + 4) - 3(x + 4) = (x + 4)(2x - 3)$

b. The binomials $y - 2$ and $2 - y$ are opposites. Factor -1 from $2 - y$ to obtain a common binomial factor.

$3y^2(y - 2) + 5(2 - y) = 3y^2(y - 2) - 5(y - 2)$ Factor -1 from $(2 - y)$.

$= (y - 2)(3y^2 - 5)$ Distributive property

GROUPING You may be able to use the distributive property to factor polynomials with four terms. Factor a common monomial from pairs of terms, then look for a common binomial factor. This is called **factor by grouping**.

EXAMPLE 2 Factor by grouping

Factor the polynomial.

a. $x^3 + 3x^2 + 5x + 15$ **b.** $y^2 + y + yx + x$

Solution

a. $x^3 + 3x^2 + 5x + 15 = (x^3 + 3x^2) + (5x + 15)$ Group terms.

$= x^2(x + 3) + 5(x + 3)$ Factor each group.

$= (x + 3)(x^2 + 5)$ Distributive property

b. $y^2 + y + yx + x = (y^2 + y) + (yx + x)$ Group terms.

$= y(y + 1) + x(y + 1)$ Factor each group.

$= (y + 1)(y + x)$ Distributive property

CHECK WORK Remember that you can check a factorization by multiplying the factors.

EXAMPLE 3 Factor by grouping

Factor $x^3 - 6 + 2x - 3x^2$.

Solution

The terms x^2 and -6 have no common factor. Use the commutative property to rearrange the terms so that you can group terms with a common factor.

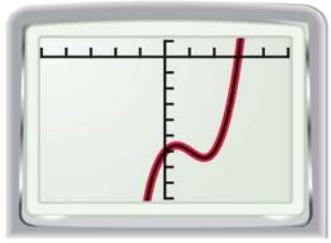
$$x^3 - 6 + 2x - 3x^2 = x^3 - 3x^2 + 2x - 6 \quad \text{Rearrange terms.}$$

$$= (x^3 - 3x^2) + (2x - 6) \quad \text{Group terms.}$$

$$= x^2(x - 3) + 2(x - 3) \quad \text{Factor each group.}$$

$$= (x - 3)(x^2 + 2) \quad \text{Distributive property}$$

CHECK Check your factorization using a graphing calculator. Graph $y_1 = x^3 - 6 + 2x - 3x^2$ and $y_2 = (x - 3)(x^2 + 2)$. Because the graphs coincide, you know that your factorization is correct.



✓ GUIDED PRACTICE for Examples 1, 2, and 3

Factor the expression.

1. $x(x - 2) + (x - 2)$ **2.** $a^3 + 3a^2 + a + 3$ **3.** $y^2 + 2x + yx + 2y$

READING
If a polynomial has two or more terms and is unfactorable, it is called a *prime polynomial*.

FACTORING COMPLETELY You have seen that the polynomial $x^2 - 1$ can be factored as $(x + 1)(x - 1)$. This polynomial is factorable. Notice that the polynomial $x^2 + 1$ cannot be written as the product of polynomials with integer coefficients. This polynomial is unfactorable. A factorable polynomial with integer coefficients is **factored completely** if it is written as a product of unfactorable polynomials with integer coefficients.

CONCEPT SUMMARY *For Your Notebook*

Guidelines for Factoring Polynomials Completely

To factor a polynomial completely, you should try each of these steps.

1. Factor out the greatest common monomial factor. *(Lesson 9.4)*	$3x^2 + 6x = 3x(x + 2)$
2. Look for a difference of two squares or a perfect square trinomial. *(Lesson 9.7)*	$x^2 + 4x + 4 = (x + 2)^2$
3. Factor a trinomial of the form $ax^2 + bx + c$ into a product of binomial factors. *(Lessons 9.5 and 9.6)*	$3x^2 - 5x - 2 = (3x + 1)(x - 2)$
4. Factor a polynomial with four terms by grouping. *(Lesson 9.8)*	$x^3 + x - 4x^2 - 4 = (x^2 + 1)(x - 4)$

EXAMPLE 4 Factor completely

Factor the polynomial completely.

a. $n^2 + 2n - 1$ **b.** $4x^3 - 44x^2 + 96x$ **c.** $50h^4 - 2h^2$

Solution

a. The terms of the polynomial have no common monomial factor. Also, there are no factors of -1 that have a sum of 2. This polynomial cannot be factored.

b. $4x^3 - 44x^2 + 96x = 4x(x^2 - 11x + 24)$ Factor out $4x$.

$= 4x(x - 3)(x - 8)$ Find two negative factors of 24 that have a sum of -11.

c. $50h^4 - 2h^2 = 2h^2(25h^2 - 1)$ Factor out $2h^2$.

$= 2h^2(5h - 1)(5h + 1)$ Difference of two squares pattern

GUIDED PRACTICE for Example 4

Factor the polynomial completely.

4. $3x^3 - 12x$ **5.** $2y^3 - 12y^2 + 18y$ **6.** $m^3 - 2m^2 - 8m$

EXAMPLE 5 Solve a polynomial equation

Solve $3x^3 + 18x^2 = -24x$.

$3x^3 + 18x^2 = -24x$ Write original equation.

$3x^3 + 18x^2 + 24x = 0$ Add $24x$ to each side.

$3x(x^2 + 6x + 8) = 0$ Factor out $3x$.

$3x(x + 2)(x + 4) = 0$ Factor trinomial.

$3x = 0$ *or* $x + 2 = 0$ *or* $x + 4 = 0$ Zero-product property

$x = 0$ $x = -2$ $x = -4$ Solve for x.

▸ The solutions of the equation are 0, -2, and -4.

CHECK Check each solution by substituting it for x in the equation. One check is shown here.

$3(-2)^3 + 18(-2)^2 \stackrel{?}{=} -24(-2)$

$-24 + 72 \stackrel{?}{=} 48$

$48 = 48$ ✓

GUIDED PRACTICE for Example 5

Solve the equation.

7. $w^3 - 8w^2 + 16w = 0$ **8.** $x^3 - 25x = 0$ **9.** $c^3 - 7c^2 + 12c = 0$

EXAMPLE 6 Solve a multi-step problem

TERRARIUM A terrarium in the shape of a rectangular prism has a volume of 4608 cubic inches. Its length is more than 10 inches. The dimensions of the terrarium are shown. Find the length, width, and height of the terrarium.

Solution

STEP 1 **Write** a verbal model. Then write an equation.

Volume (cubic inches)	=	Length (inches)	·	Width (inches)	·	Height (inches)
4608	=	$(36 - w)$	·	w	·	$(w + 4)$

STEP 2 **Solve** the equation for w.

$4608 = (36 - w)(w)(w + 4)$ — Write equation.

$0 = 32w^2 + 144w - w^3 - 4608$ — Multiply. Subtract 4608 from each side.

$0 = (-w^3 + 32w^2) + (144w - 4608)$ — Group terms.

$0 = -w^2(w - 32) + 144(w - 32)$ — Factor each group.

$0 = (w - 32)(-w^2 + 144)$ — Distributive property

$0 = -1(w - 32)(w^2 - 144)$ — Factor -1 from $-w^2 + 144$.

$0 = -1(w - 32)(w - 12)(w + 12)$ — Difference of two squares pattern

$w - 32 = 0$ *or* $w - 12 = 0$ *or* $w + 12 = 0$ — Zero-product property

$w = 32$ $\quad w = 12$ $\quad w = -12$ — Solve for w.

STEP 3 **Choose** the solution of the equation that is the correct value of w. Disregard $w = -12$, because the width cannot be negative.

You know that the length is more than 10 inches. Test the solutions 12 and 32 in the expression for the length.

Length $= 36 - 12 = 24$ ✓ *or* Length $= 36 - 32 = 4$ ✗

The solution 12 gives a length of 24 inches, so 12 is the correct value of w.

STEP 4 **Find** the height.

Height $= w + 4 = 12 + 4 = 16$

▶ The width is 12 inches, the length is 24 inches, and the height is 16 inches.

GUIDED PRACTICE for Example 6

10. **DIMENSIONS OF A BOX** A box in the shape of a rectangular prism has a volume of 72 cubic feet. The box has a length of x feet, a width of $(x - 1)$ feet, and a height of $(x + 9)$ feet. Find the dimensions of the box.

9.8 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 13, 23, and 71

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 41, 55, 71, and 73

SKILL PRACTICE

1. **VOCABULARY** What does it mean for a polynomial to be factored completely?

2. ★ **WRITING** *Explain* how you know if a polynomial is unfactorable.

EXAMPLE 1 on p. 606 for Exs. 3–12

BINOMIAL FACTORS **Factor the expression.**

3. $x(x - 8) + (x - 8)$
4. $5y(y + 3) - 2(y + 3)$
5. $6z(z - 4) - 7(z - 4)$
6. $10(a - 6) - 3a(a - 6)$
7. $b^2(b + 5) - 3(b + 5)$
8. $7c^2(c + 9) + 2(c + 9)$
9. $x(13 + x) - (x + 13)$
10. $y^2(y - 4) + 5(4 - y)$
11. $12(z - 1) - 5z^2(1 - z)$

12. ★ **MULTIPLE CHOICE** Which is the correct factorization of $x^2(x - 8) + 5(8 - x)$?

Ⓐ $(x^2 + 5)(x - 8)$ Ⓑ $(x^2 + 5)(8 - x)$

Ⓒ $(x^2 - 5)(x - 8)$ Ⓓ $(x^2 - 5)(8 - x)$

EXAMPLES 2 and 3 on pp. 606–607 for Exs. 13–22

FACTORING BY GROUPING **Factor the polynomial.**

13. $x^3 + x^2 + 2x + 2$
14. $y^3 - 9y^2 + y - 9$
15. $z^3 - 4z^2 + 3z - 12$
16. $c^3 + 7c^2 + 5c + 35$
17. $a^3 + 13a^2 - 5a - 65$
18. $2s^3 - 3s^2 + 18s - 27$
19. $5n^3 - 4n^2 + 25n - 20$
20. $x^2 + 8x - xy - 8y$
21. $y^2 + y + 5xy + 5x$

22. **ERROR ANALYSIS** *Describe* and correct the error in factoring.

$$a^3 + 8a^2 - 6a - 48 = a^2(a + 8) + 6(a + 8)$$
$$= (a + 8)(a^2 + 6)$$

EXAMPLE 4 on p. 608 for Exs. 23–42

FACTORING COMPLETELY **Factor the polynomial completely.**

23. $x^4 - x^2$
24. $36a^4 - 4a^2$
25. $3n^5 - 48n^3$
26. $4y^6 - 16y^4$
27. $75c^9 - 3c^7$
28. $72p - 2p^3$
29. $32s^4 - 8s^2$
30. $80z^8 - 45z^6$
31. $m^2 - 5m - 35$
32. $6g^3 - 24g^2 + 24g$
33. $3w^4 + 24w^3 + 48w^2$
34. $3r^5 + 3r^4 - 90r^3$
35. $b^3 - 5b^2 - 4b + 20$
36. $h^3 + 4h^2 - 25h - 100$
37. $9t^3 + 18t - t^2 - 2$
38. $2x^5y - 162x^3y$
39. $7a^3b^3 - 63ab^3$
40. $-4s^3t^3 + 24s^2t^2 - 36st$

41. ★ **MULTIPLE CHOICE** What is the completely factored form of $3x^6 - 75x^4$?

Ⓐ $3x^4(x^2 - 25)$ Ⓑ $3x^4(x - 5)^2$ Ⓒ $3x^4(x + 5)^2$ Ⓓ $3x^4(x - 5)(x + 5)$

42. **ERROR ANALYSIS** *Describe* and correct the error in factoring the polynomial completely.

$$x^3 - 6x^2 - 9x + 54 = x^2(x - 6) - 9(x - 6)$$
$$= (x - 6)(x^2 - 9)$$

EXAMPLE 5 on p. 608 for Exs. 43–54

SOLVING EQUATIONS **Solve the equation.**

43. $x^3 + x^2 - 4x - 4 = 0$ **44.** $a^3 - 11a^2 - 9a + 99 = 0$ **45.** $4y^3 - 7y^2 - 16y + 28 = 0$

46. $5n^3 - 30n^2 + 40n = 0$ **47.** $3b^3 + 24b^2 + 45b = 0$ **48.** $2t^5 + 2t^4 - 144t^3 = 0$

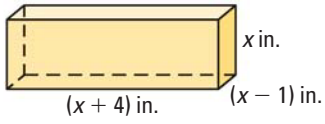
49. $z^3 - 81z = 0$ **50.** $c^4 - 100c^2 = 0$ **51.** $12s - 3s^3 = 0$

52. $2x^3 - 10x^2 + 40 = 8x$ **53.** $3p + 1 = p^2 + 3p^3$ **54.** $m^3 - 3m^2 = 4m - 12$

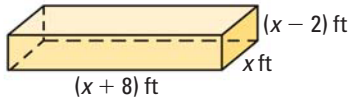
55. ★ **WRITING** Is it possible to find three solutions of the equation $x^3 + 2x^2 + 3x + 6 = 0$? *Explain* why or why not.

GEOMETRY **Find the length, width, and height of the rectangular prism with the given volume.**

56. Volume = 12 cubic inches



57. Volume = 96 cubic feet



FACTORING COMPLETELY **Factor the polynomial completely.**

58. $x^3 + 2x^2y - x - 2y$ **59.** $8b^3 - 4b^2a - 18b + 9a$ **60.** $4s^2 - s + 12st - 3t$

FACTOR BY GROUPING **In Exercises 61–66, use the example below to factor the trinomial by grouping.**

EXAMPLE **Factor a trinomial by grouping**

Factor $8x^2 + 10x - 3$ by grouping.

Solution

Notice that the polynomial is in the form $ax^2 + bx + c$.

STEP 1 **Write** the product ac as the product of two factors that have a sum of b. In this case, the product ac is $8(-3) = -24$. Find two factors of -24 that have a sum of 10.

$-24 = 12 \cdot (-2)$ and $12 + (-2) = 10$

STEP 2 **Rewrite** the middle term as two terms with coefficients 12 and -2.

$8x^2 + \mathbf{10x} - 3 = 8x^2 \mathbf{+ 12x - 2x} - 3$

STEP 3 **Factor** by grouping.

$8x^2 + 12x - 2x - 3 = (8x^2 + 12x) + (-2x - 3)$ **Group terms.**

$= 4x(2x + 3) - (2x + 3)$ **Factor each group.**

$= (2x + 3)(4x - 1)$ **Distributive property**

61. $6x^2 + 5x - 4$ **62.** $10s^2 + 19s + 6$ **63.** $12n^2 - 13n + 3$

64. $16a^2 + 14a + 3$ **65.** $21w^2 + 8w - 4$ **66.** $15y^2 - 31y + 10$

67. **CHALLENGE** Use factoring by grouping to show that a trinomial of the form $a^2 + 2ab + b^2$ can be factored as $(a + b)^2$. *Justify* your steps.

PROBLEM SOLVING

EXAMPLE 6 on p. 609 for Exs. 68–70

68. CYLINDRICAL VASE A vase in the shape of a cylinder has a height of 6 inches and a volume of 24π cubic inches. What is the radius of the vase?

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69. CARPENTRY You are building a birdhouse that will have a volume of 128 cubic inches. The birdhouse will have the dimensions shown.

a. Write a polynomial that represents the volume of the birdhouse.

b. What are the dimensions of the birdhouse?

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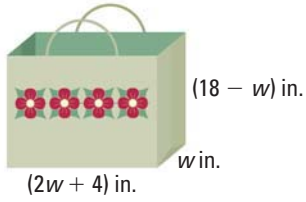
70. BAG SIZE A gift bag is shaped like a rectangular prism and has a volume of 1152 cubic inches. The dimensions of the gift bag are shown. The height is greater than the width. What are the dimensions of the gift bag?

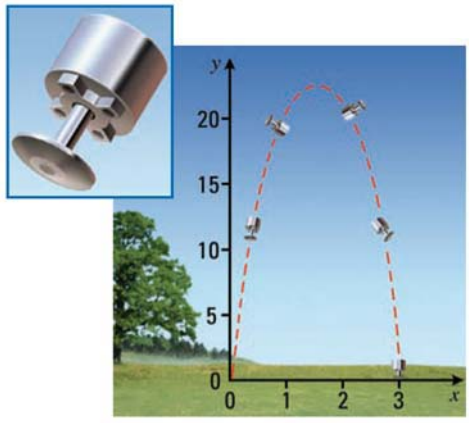


71. ★ SHORT RESPONSE A pallino is the small target ball that is tossed in the air at the beginning of a game of bocce. The height h (in meters) of the pallino after you throw it can be modeled by $h = -4.9t^2 + 3.9t + 1$ where t is the time (in seconds) since you released it.

a. Find the zeros of the function.

b. Do the zeros of the function have any meaning in this situation? *Explain* your reasoning.

72. JUMPING ROBOT The path of a jumping robot can be modeled by the graph of the equation $y = -10x^2 + 30x$ where x and y are both measured in feet. On a coordinate plane, the ground is represented by the x-axis, and the robot's starting position is the origin.



a. The robot's maximum height is 22.5 feet. What is the robot's horizontal distance from its starting point when its height is 22.5 feet?

b. How far has the robot traveled horizontally when it lands on the ground? *Explain* your answer.

73. ★ EXTENDED RESPONSE The width of a box is 4 inches more than the height h. The length is the difference of 9 inches and the height.

a. Write a polynomial that represents the volume of the box.

b. The volume of the box is 180 cubic inches. What are all the possible dimensions of the box?

c. Which dimensions result in a box with the smallest possible surface area? *Explain* your reasoning.

○ = WORKED-OUT SOLUTIONS only: Exs. 71

★ = STANDARDIZED TEST PRACTICE

74. CHALLENGE A plastic cube is used to display an autographed baseball. The cube has an outer surface area of 54 square inches.

a. What is the length of an outer edge of the cube?

b. What is the greatest volume the cube can possibly have? *Explain* why the actual volume inside of the cube may be less than the greatest possible volume.

MIXED REVIEW

PREVIEW
Prepare for Lesson 10.1 in Exs. 75–86.

Graph the equation. *(p. 244)*

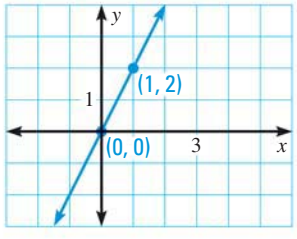
75. $y - 2x = 0$ **76.** $y + 2x = 3$ **77.** $y + 5x = 2$ **78.** $2y - 6x = 6$

79. $-3y + 4x = 12$ **80.** $-4x + 2y = 8$ **81.** $x - 4y = 2$ **82.** $x - 2y = -10$

83. $y = 5$ **84.** $y = 0$ **85.** $x = -4$ **86.** $x = 2$

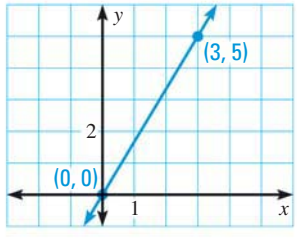
Write an equation of the line shown.

87.



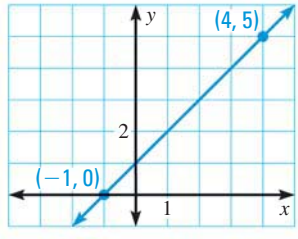
(p. 283)

88.



(p. 283)

89.



(p. 283)

QUIZ *for Lessons 9.7–9.8*

Factor the polynomial. *(p. 600)*

1. $x^2 - 400$ **2.** $18 - 32z^2$ **3.** $169x^2 - 25y^2$

4. $n^2 - 6n + 9$ **5.** $100a^2 + 20a + 1$ **6.** $8r^2 - 40rs + 50s^2$

Factor the polynomial completely. *(p. 606)*

7. $3x^5 - 75x^3$ **8.** $72s^4 - 8s^2$ **9.** $3x^4y - 300x^2y$

10. $a^3 - 4a^2 - 21a$ **11.** $2h^4 + 28h^3 + 98h^2$ **12.** $z^3 - 4z^2 - 16z + 64$

Solve the equation.

13. $x^2 + 10x + 25 = 0$ *(p. 600)* **14.** $48 - 27m^2 = 0$ *(p. 600)*

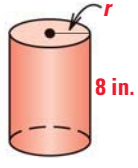
15. $w^3 - w^2 - 4w + 4 = 0$ *(p. 606)* **16.** $4x^3 - 28x^2 + 40x = 0$ *(p. 606)*

17. $3x^5 - 6x^4 - 45x^3 = 0$ *(p. 606)* **18.** $x^3 - 121x = 0$ *(p. 606)*

19. VOLUME The cylinder shown has a volume of 72π cubic inches. *(p. 600)*

a. Write a polynomial that represents the volume of the cylinder. Leave your answer in terms of π.

b. Find the radius of the cylinder.



MIXED REVIEW of Problem Solving

STATE TEST PRACTICE
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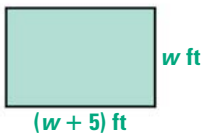
Lessons 9.5–9.8

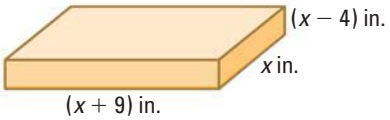
1. MULTI-STEP PROBLEM A rectangular room has the dimensions shown.



a. Write a polynomial that represents the area of the room.

b. The room has an area of 150 square feet. What are the length and width of the room?

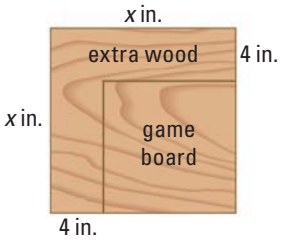
2. MULTI-STEP PROBLEM A block of clay has the dimensions shown.



a. Write a polynomial that represents the volume of the clay.

b. The clay has a volume of 180 cubic inches. What are the length, width, and height of the block?

3. MULTI-STEP PROBLEM You are making a wooden game board. You cut a square piece of wood, as shown.



a. Write a polynomial that represents the area of the game board.

b. The area of the game board is 100 square inches. What was the area of the original piece of wood? *Explain* how you found your answer.

4. OPEN-ENDED *Describe* a situation that can be modeled using the vertical motion model $h = -16t^2 + 48t$. Then find the value of t when $h = 0$. *Explain* what this value of t means in this situation.

5. EXTENDED RESPONSE You hit a baseball straight up into the air. The baseball is hit with an initial vertical velocity of 80 feet per second when it is 3 feet off the ground.

a. Write an equation that gives the height (in feet) of the baseball as a function of the time (in seconds) since it was hit.

b. After how many seconds does the ball reach a height of 99 feet?

c. Does the ball reach a height of 99 feet more than once? *Justify* your answer.

6. EXTENDED RESPONSE The length of a box is 25 inches more than its height. The width of the box is 1 inch less than its height.

a. Draw a diagram of the box. Label its dimensions in terms of the height h.

b. Write a polynomial that represents the volume of the box.

c. The box has a volume of 600 cubic inches. What is the area of its top? *Explain*.

7. SHORT RESPONSE A tennis player hits a ball with an initial vertical velocity of 63 feet per second. Can you find the number of seconds the tennis ball is in the air? *Explain* why not or find the number of seconds.

8. GRIDDED ANSWER During an experiment in physics class, you drop a ball from a height of 144 feet. After how many seconds does the ball hit the ground?

9. SHORT RESPONSE A football is kicked toward a goal post that is 10 feet high. The path of the football is modeled by the graph of $y = -0.005x^2 + 0.6x$ where x and y are measured in feet. On a coordinate plane, the x-axis represents the ground, and the ball leaves the ground at the origin. The ball hits the goal post on the way down. How far from the goal post is the kicker? *Explain*.

9 CHAPTER SUMMARY

BIG IDEAS

For Your Notebook

Big Idea 1 Adding, Subtracting, and Multiplying Polynomials

You can perform operations with polynomials using the steps below.

Operation	Steps
Add	Group like terms and add.
Subtract	First, rewrite subtraction as addition. Second, group like terms and add.
Multiply	First, multiply terms using the distributive property. Second, combine like terms.

Big Idea 2 Factoring Polynomials

When factoring a polynomial, you should use the following checklist so that you can be sure you have factored the polynomial completely.

STEP 1 **Factor** out the greatest common monomial factor.

STEP 2 **Look** for special products to factor.

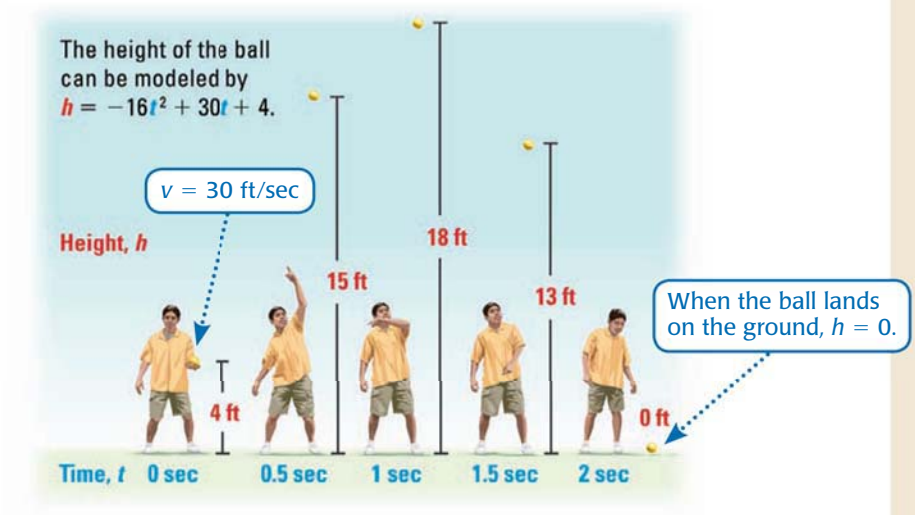
STEP 3 **Factor** a trinomial into a pair of binomials, if possible.

STEP 4 **Factor** a polynomial with four terms by grouping, if possible.

Big Idea 3 Writing and Solving Polynomial Equations to Solve Problems

You can write polynomials that model real-world situations in order to solve problems. For example, you can use the vertical motion model.

Height (in feet) of a projectile: $h = -16t^2 + vt + s$ where t is the time (in seconds) the object has been in the air, v is the initial vertical velocity (in feet per second), and s is the initial height (in feet).



9 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

- monomial, *p. 554*
- degree of a monomial, *p. 554*
- polynomial, *p. 554*
- degree of a polynomial, *p. 554*
- leading coefficient, *p. 554*
- binomial, *p. 555*
- trinomial, *p. 555*
- roots, *p. 575*
- vertical motion model, *p. 577*
- perfect square trinomial, *p. 601*
- factor by grouping, *p. 606*
- factor completely, *p. 607*

VOCABULARY EXERCISES

1. Copy and complete: The greatest degree of the terms in a polynomial is called the ___?___.

2. **WRITING** Is $2x^{-1}$ a monomial? *Explain* why or why not.

3. **WRITING** What does it mean for a polynomial to be factored completely? Give an example of a polynomial that has been factored completely.

In Exercises 4–6, match the polynomial with its classification.

4. $5x - 22$ **5.** $-11x^3$ **6.** $x^2 + x + 1$

A. Monomial **B.** Binomial **C.** Trinomial

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 9.

9.1 Add and Subtract Polynomials *pp. 554–559*

EXAMPLE

Find the difference $(3x^2 + 2) - (4x^2 - x - 9)$.

Use a vertical format.

$$\begin{array}{rr} & 3x^2 + 2 \\ - & (4x^2 - x - 9) \end{array} \quad \rightarrow \quad \begin{array}{rr} & 3x^2 + 2 \\ + & -4x^2 + x + 9 \\ \hline & -x^2 + x + 11 \end{array}$$

EXERCISES

EXAMPLES 3 and 4 on pp. 555–556 for Exs. 7–12

Find the sum or difference.

7. $(9x + 6x^3 - 8x^2) + (-5x^3 + 6x)$

8. $(7a^3 - 4a^2 - 2a + 1) + (a^3 - 1)$

9. $(11y^5 + 3y^2 - 4) + (y^2 - y + 1)$

10. $(3n^2 - 4n + 1) - (8n^2 - 4n + 17)$

11. $(2s^3 + 8) - (-3s^3 + 7s - 5)$

12. $(-k^2 + 7k + 5) - (2k^4 - 3k^3 - 6)$

9.2 Multiply Polynomials

pp. 562–568

EXAMPLE

Find the product.

a. $(x^2 + 4x - 5)(2x - 1)$ **b.** $(5y + 6)(y - 3)$

Solution

a. Use a horizontal format.

$(x^2 + 4x - 5)(2x - 1)$ **Write product.**

$= x^2(2x - 1) + 4x(2x - 1) - 5(2x - 1)$ **Distributive property**

$= 2x^3 - x^2 + 8x^2 - 4x - 10x + 5$ **Distributive property**

$= 2x^3 + 7x^2 - 14x + 5$ **Combine like terms.**

b. Use a vertical format.

STEP 1 **Multiply** by -3.

$$\begin{array}{rr} & 5y + 6 \\ \times & y - 3 \\ \hline & -15y - 18 \end{array}$$

STEP 2 **Multiply** by y.

$$\begin{array}{rr} & 5y + 6 \\ \times & y - 3 \\ \hline & -15y - 18 \\ 5y^2 + 6y & \end{array}$$

STEP 3 **Add** products.

$$\begin{array}{rr} & 5y + 6 \\ \times & y - 3 \\ \hline & -15y - 18 \\ 5y^2 + 6y & \\ \hline 5y^2 - 9y - 18 & \end{array}$$

EXERCISES

EXAMPLES 1, 2, 3, and 4 on pp. 562–563 for Exs. 13–21

Find the product.

13. $(x^2 - 2x + 1)(x - 3)$ **14.** $(y^2 + 5y + 4)(3y + 2)$ **15.** $(x - 4)(x + 2)$

16. $(5b^2 - b - 7)(b + 6)$ **17.** $(z + 8)(z - 11)$ **18.** $(2a - 1)(a - 3)$

19. $(6n + 7)(3n + 1)$ **20.** $(4n - 5)(7n - 3)$ **21.** $(3x - 2)(x + 4)$

9.3 Find Special Products of Polynomials

pp. 569–574

EXAMPLE

Find the product $(3x + 2)(3x - 2)$.

$(3x + 2)(3x - 2) = (3x)^2 - 2^2$ **Sum and difference pattern**

$= 9x^2 - 4$ **Simplify.**

EXERCISES

EXAMPLES 1 and 2 on pp. 569–570 for Exs. 22–27

Find the product.

22. $(x + 11)^2$ **23.** $(6y + 1)^2$ **24.** $(2x - y)^2$

25. $(4a - 3)^2$ **26.** $(k + 7)(k - 7)$ **27.** $(3s + 5)(3s - 5)$

9 CHAPTER REVIEW

9.4 Solve Polynomial Equations in Factored Form

pp. 575–580

EXAMPLE

Solve $6x^2 + 42x = 0$.

$6x^2 + 42x = 0$ — Write original equation.

$6x(x + 7) = 0$ — Factor left side.

$6x = 0$ *or* $x + 7 = 0$ — Zero-product property

$x = 0$ *or* $x = -7$ — Solve for x.

▶ The solutions of the equation are 0 and -7.

EXERCISES

EXAMPLES 3 and 4 on p. 576 for Exs. 28–33

Solve the equation.

28. $2a^2 + 26a = 0$

29. $3t^2 - 33t = 0$

30. $8x^2 - 4x = 0$

31. $m^2 = 9m$

32. $5y^2 = -50y$

33. $21h^2 = 7h$

9.5 Factor $x^2 + bx + c$

pp. 583–589

EXAMPLE

Factor $x^2 + 2x - 63$.

Find two factors of -63 whose sum is 2. One factor will be positive, and the other will be negative. Make an organized list of factors.

Factors of −63	Sum of factors	
1, −63	1 + (−63) = −62	✗
−1, 63	−1 + 63 = 62	✗
3, −21	3 + (−21) = −18	✗
−3, 21	−3 + 21 = 18	✗
9, −7	9 + (−7) = 2	← Correct sum
−9, 7	−9 + 7 = −2	✗

▶ $x^2 + 2x - 63 = (x + 9)(x - 7)$

EXERCISES

EXAMPLES 1,2 and 3 on pp. 583–584 for Exs. 34–42

Factor the trinomial.

34. $n^2 + 15n + 26$

35. $s^2 + 10s - 11$

36. $b^2 - 5b - 14$

37. $a^2 + 5a - 84$

38. $t^2 - 24t + 135$

39. $x^2 + 4x - 32$

40. $p^2 + 9p + 14$

41. $c^2 + 8c + 15$

42. $y^2 - 10y + 21$

9.6 Factor $ax^2 + bx + c$

pp. 593–599

EXAMPLE

THROWN BALL You throw a ball up into the air. At 4 feet above the ground, the ball leaves your hand with an initial vertical velocity of 30 feet per second.

a. Write an equation that gives the height (in feet) of the ball as a function of the time (in seconds) since it left your hand.

b. After how many seconds does the ball land on the ground?

Solution

a. Use the vertical motion model $h = -16t^2 + vt + s$ to write an equation for the height h (in feet) of the ball as a function of the time t (in seconds). In this case, $v = 30$ and $s = 4$.

$h = -16t^2 + vt + s$ — **Vertical motion model**

$h = -16t^2 + 30t + 4$ — **Substitute 30 for *v* and 4 for *s*.**

b. When the ball lands on the ground, its height is 0 feet. Substitute 0 for h and solve the equation for t.

$0 = -16t^2 + 30t + 4$ — **Substitute 0 for *h*.**

$0 = -2(8t^2 - 15t - 2)$ — **Factor out −2.**

$0 = -2(8t + 1)(t - 2)$ — **Factor the trinomial. Find factors of 8 and −2 that produce a middle term with a coefficient of −15.**

$8t + 1 = 0$ *or* $t - 2 = 0$ — **Zero-product property**

$t = -\frac{1}{8}$ *or* $t = 2$ — **Solve for *t*.**

The solutions of the equation are $-\frac{1}{8}$ and 2. A negative solution does not make sense in this situation, so disregard $-\frac{1}{8}$.

▶ The ball lands on the ground after 2 seconds.

EXERCISES

EXAMPLES 1, 2, 3, and 4 on pp. 593–595 for Exs. 43–50

Solve the equation.

43. $7x^2 - 8x = -1$

44. $4n^2 + 3 = 7n$

45. $3s^2 + 4s + 4 = 8$

46. $6z^2 + 13z = 5$

47. $-4r^2 = 18r + 18$

48. $9a^2 = 6a + 24$

49. THROWN BALL You throw a ball up into the air with an initial vertical velocity of 46 feet per second. The ball leaves your hand when it is 6 feet above the ground. After how many seconds does the ball land on the ground?

50. GEOMETRY The length of a rectangle is 1 inch less than twice the width. The area of the rectangle is 21 square inches. What is the length of the rectangle?

9 CHAPTER REVIEW

9.7 Factor Special Products

pp. 600–605

EXAMPLE

Factor the polynomial.

a. $100x^2 - y^2$ **b.** $4x^2 - 36x + 81$

Solution

a. $100x^2 - y^2 = (10x)^2 - y^2$ — Write as $a^2 - b^2$.

$= (10x + y)(10x - y)$ — Difference of two squares pattern

b. $4x^2 - 36x + 81 = (2x)^2 - 2(2x \cdot 9) + 9^2$ — Write as $a^2 - 2ab + b^2$.

$= (2x - 9)^2$ — Perfect square trinomial pattern

EXERCISES

EXAMPLES 1, 2, 3, 4, and 6 on pp. 600–602 for Exs. 51–57

Factor the polynomial.

51. $z^2 - 225$ **52.** $a^2 - 16y^2$ **53.** $12 - 48n^2$

54. $x^2 + 20x + 100$ **55.** $16p^2 - 8p + 1$ **56.** $-2y^2 + 32y - 128$

57. DROPPED OBJECT You drop a penny from a height of 16 feet. After how many seconds does the penny land on the ground?

9.8 Factor Polynomials Completely

pp. 606–613

EXAMPLE

Factor the polynomial completely.

a. $y^3 - 4y^2 + 8y - 32$ **b.** $5x^3 - 40x^2 + 80x$

Solution

a. $y^3 - 4y^2 + 8y - 32 = (y^3 - 4y^2) + (8y - 32)$ — Group terms.

$= y^2(y - 4) + 8(y - 4)$ — Factor each group.

$= (y - 4)(y^2 + 8)$ — Distributive property

b. $5x^3 - 40x^2 + 80x = 5x(x^2 - 8x + 16)$ — Factor out $5x$.

$= 5x(x - 4)^2$ — Perfect square trinomial pattern

EXERCISES

EXAMPLE 4 on pp. 606–608 for Exs. 58–66

Factor the polynomial completely.

58. $a^3 + 6a - 5a^2 - 30$ **59.** $y^2 + 3y + yx + 3x$ **60.** $x^3 - 11x^2 - x + 11$

61. $5s^4 - 125s^2$ **62.** $147n^5 - 3n^3$ **63.** $2z^3 + 2z^2 - 60z$

64. $x^3 + 5x^2 - x - 5$ **65.** $2b^3 + 3b^2 - 8b - 12$ **66.** $x^3 + x^2 - 6x - 6$

9 CHAPTER TEST

Find the sum or difference.

1. $(a^2 - 4a + 6) + (-3a^2 + 13a + 1)$

2. $(5x^2 - 2) + (8x^3 + 2x^2 - x + 9)$

3. $(15n^2 + 7n - 1) - (4n^2 - 3n - 8)$

4. $(9c^3 - 11c^2 + 2c) - (-6c^2 - 3c + 11)$

Find the product.

5. $(2z + 9)(z - 7)$

6. $(5m - 8)(5m - 7)$

7. $(b + 2)(-b^2 + 4b - 3)$

8. $(5 + 7y)(1 - 9y)$

9. $(2x^2 - 3x + 5)(x - 4)$

10. $(5p - 6)(5p + 6)$

11. $(12 - 3g)^2$

12. $(2s + 9t)^2$

13. $(11a - 4b)(11a + 4b)$

Factor the polynomial.

14. $x^2 + 8x + 7$

15. $2n^2 - 11n + 15$

16. $-12r^2 + 5r + 3$

17. $t^2 - 10t + 25$

18. $-3n^2 + 75$

19. $3x^2 + 29x - 44$

20. $x^2 - 49$

21. $2a^4 + 21a^3 + 49a^2$

22. $y^3 + 2y^2 - 81y - 162$

Solve the equation.

23. $25a = 10a^2$

24. $21z^2 + 85z - 26 = 0$

25. $x^2 - 22x = -121$

26. $a^2 - 11a + 24 = 0$

27. $t^2 + 7t = 60$

28. $4x^2 = 22x + 42$

29. $56b^2 + b = 1$

30. $n^3 - 121n = 0$

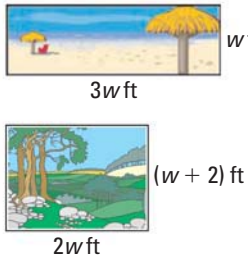
31. $a^3 + a^2 = 64a + 64$

32. VERTICAL MOTION A cricket jumps off the ground with an initial vertical velocity of 4 feet per second.

a. Write an equation that gives the height (in feet) of the cricket as a function of the time (in seconds) since it jumps.

b. After how many seconds does the cricket land on the ground?

33. POSTER AREA Two posters have the lengths and widths shown. The posters have the same area.

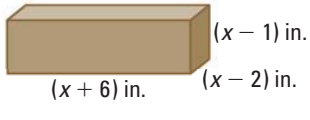


a. Write an equation that relates the areas of the two posters.

b. Find the length and width of each poster.

34. CONSTRUCTION A construction worker is working on the roof of a building. A drop of paint falls from a rafter that is 225 feet above the ground. After how many seconds does the paint hit the ground?

35. BOX DIMENSIONS A cardboard box that is a rectangular prism has the dimensions shown.



a. Write a polynomial that represents the volume of the box.

b. The volume of the box is 60 cubic inches. What are the length, width, and height of the box?

9 ★ Standardized TEST PREPARATION

SHORT RESPONSE QUESTIONS

Scoring Rubric

Full Credit
- solution is complete and correct

Partial Credit
- solution is complete but errors are made,

or
- solution is without error but incomplete

No Credit
- no solution is given,

or
- solution makes no sense

PROBLEM

A rectangular photo has an area of 24 square inches. You trim the photo so that it fits into a square frame. You trim 3 inches from the length and 1 inch from the width of the photo. Write and solve an equation to find the side length of the resulting square photo. *Explain* how you chose one solution of the equation to find the side length.

Below are sample solutions to the problem. Read each solution and the comments in blue to see why the sample represents full credit, partial credit, or no credit.

SAMPLE 1: Full credit solution

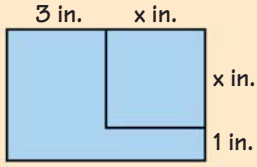
Draw a diagram. Use the formula for the area of a rectangle.

A diagram shows how the equation is obtained.

$A = \ell \cdot w$

$= (x + 3)(x + 1)$

$= x^2 + 4x + 3$



The correct calculations are performed.

Substitute 24 for A and solve.

$24 = x^2 + 4x + 3$

$0 = x^2 + 4x - 21$

$0 = (x - 3)(x + 7)$

$0 = x - 3$ *or* $0 = x + 7$

$x = 3$ *or* $x = -7$

The question is answered correctly and includes an explanation.

A solution represents the side length of the square photo. A negative side length does not make sense, so choose $x = 3$. The side length is 3 inches.

SAMPLE 2: Partial credit solution

The length of the rectangular photo is $x + 3$. The width is $x + 1$.

The equation is correct, and the student has explained how it was obtained.

$A = (x + 3)(x + 1)$

$24 = x^2 + 4x + 3$

$0 = x^2 + 4x - 21$

$0 = (x - 3)(x + 7)$

The question is answered correctly but does not include an explanation.

$x = 3$ *or* $x = -7$

The side length of the square photo is 3 inches.

SAMPLE 3: Partial credit solution

The equation and its solutions are correct, but the student found the length and width of the original photo instead of the trimmed photo.

$$24 = (x + 3)(x + 1)$$
$$0 = x^2 + 4x - 21$$
$$0 = (x - 3)(x + 7)$$
$$x = 3 \quad or \quad x = -7$$

A negative solution does not make sense in the situation. The length is $3 + 3 = 6$ inches, and the width is $3 + 1 = 4$ inches.

SAMPLE 4: No credit solution

The student's reasoning is incorrect, and the equation is incorrect. The answer is incorrect.

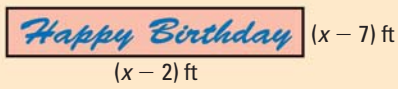
$$(x - 3)(x - 1) = 24$$
$$x^2 - 4x - 21 = 0$$
$$(x - 3)(x + 7) = 0$$

$x = -3$ or 7, so the side length of the square is 7 inches.

PRACTICE Apply the Scoring Rubric

Score the solution to the problem below as *full credit*, *partial credit*, or *no credit*. *Explain* your reasoning.

PROBLEM You are making a banner for a surprise birthday party. The banner will have the dimensions shown in the diagram. Its area will be 6 square feet. Write and solve an equation to find the length and width of the banner. *Explain* your reasoning.



1.

$$6 = (x - 2)(x - 7)$$
$$6 = x^2 - 9x + 14$$
$$0 = x^2 - 9x + 8$$
$$0 = (x - 8)(x - 1)$$
$$x = 8 \text{ or } x = 1$$

In this problem, the solutions of the equation are 8 and 1. If $x = 1$, then the width of the banner is $x - 7 = -6$. A width cannot be negative, so disregard the solution $x = 1$.

The length of the banner is $8 - 2 = 6$ feet, and the width is $8 - 7 = 1$ foot.

2.

$$6 = (x - 2)(x - 7)$$
$$6 = x^2 - 2x - 7x + 14$$
$$0 = x^2 - 9x + 20$$
$$0 = (x - 5)(x - 4)$$
$$x = 5 \text{ or } x = 4$$

The equation has two solutions. They are 5 and 4. So, the width of the banner is 5 feet, and the length of the banner is 4 feet.

9 ★ Standardized TEST PRACTICE

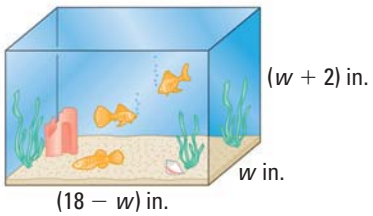
SHORT RESPONSE

1. A cat jumps straight up from the ground with an initial vertical velocity of 10 feet per second.

a. Write an equation that gives the height of the cat (in feet) as a function of the time (in seconds) since it left the ground.

b. Find the zeros of the function from part (a). *Explain* what the zeros mean in this situation.

2. A fish tank is shaped like a rectangular prism with a volume of 576 cubic inches. Its length is greater than 10 inches. The dimensions of the tank are shown in the diagram.



a. Write a polynomial that represents the volume of the fish tank.

b. Find the length, width, and height of the fish tank. *Explain* your reasoning using the solutions of the equation from part (a).

3. You throw a ball from an initial height of 5 feet and with an initial vertical velocity of 36 feet per second.

a. Write an equation that gives the height of the ball (in feet) as a function of the time (in seconds) since it left your hand.

b. How many times does the ball reach a height of 25 feet? *Explain* your reasoning using the function from part (a).

4. A pencil falls off a shelf with a height of 4 feet.

a. What is the initial vertical velocity of the pencil? *Explain* your answer.

b. After how many seconds does the pencil hit the ground?

5. During the period 1985–2000, the number S (in thousands) of students enrolled in public school in the United States and the percent p (in decimal form) of the students enrolled in public school who are also enrolled in a foreign language class can be modeled by

$$S = 27.5x^2 - 336x + 12{,}400 \text{ and}$$

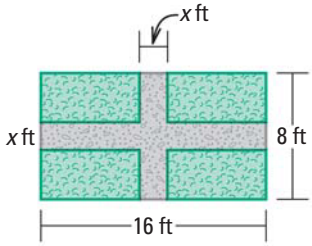
$$p = 0.008x + 0.336$$

where x is the number of years since 1985.

a. Write an equation that models the number (in thousands) of public school students in the United States enrolled in a foreign language class as a function of the number of years since 1985. *Explain* how you found this equation.

b. How many public school students in the United States were enrolled in a foreign language class in 2000?

6. Students in an environmental club are planning a garden with four rectangular plots of land separated by stone paths, as shown. The stone paths will have the same width.



a. The students plan to cover 80 square feet of path with stone. Write and solve an equation to find the width of the paths.

b. In part (a) you used one solution of an equation to find your answer. *Explain* how you chose which solution to use.

7. The shape of an entrance to a tunnel can be modeled by the graph of the equation $y = -0.2x(x - 20)$ where x and y are measured in feet. On a coordinate plane, the ground is represented by the x-axis. How wide is the tunnel at its base? *Explain* how you found your answer.

MULTIPLE CHOICE

8. Which is the correct factorization of $25x^2 - 144$?

Ⓐ $(5x + 9)(5x - 16)$

Ⓑ $(5x - 12)^2$

Ⓒ $(5x + 18)(5x - 8)$

Ⓓ $(5x - 12)(5x + 12)$

9. What are the solutions of the equation $(x + 4)(x - 12) = 0$?

Ⓐ 4 and -12 Ⓑ 4 and 12

Ⓒ -4 and -12 Ⓓ -4 and 12

10. What are the solutions of the equation $x^2 - 26 = 11x$?

Ⓐ -2 and 13 Ⓑ 2 and 13

Ⓒ -2 and -13 Ⓓ 2 and -13

GRIDDED ANSWER

11. The equation $x^2 - 20x + 100 = 0$ has two identical solutions. What is the solution of the equation?

12. The square of the binomial $x + 3$ has the form $x^2 + bx + 9$. What is the value of b?

13. What is the degree of the polynomial $6x^4 - 3x^2 + 10x$?

14. The function $f(x) = 4x^2 - 36$ has two zeros. What is the greater of the two zeros?

15. The area of a rectangle is 28 square inches. The length of the rectangle is 3 inches more than its width. What is the width (in inches) of the rectangle?

16. A pine cone falls from a tree branch that is 144 feet above the ground. After how many seconds does the pine cone land on the ground?

EXTENDED RESPONSE

17. The shape of a stone arch in a park can be modeled by the graph of the equation $y = -x^2 + 6x$ where x and y are measured in feet. On a coordinate plane, the ground is represented by the x-axis.

a. Make a table of values that shows the height of the stone arch for $x = 0, 1, 2, 3, 4$, and 5 feet.

b. Plot the ordered pairs in the table from part (a) as points in a coordinate plane. Connect the points with a smooth curve.

c. How wide is the base of the arch? *Justify* your answer using the zeros of the given function.

d. At how many points does the arch reach a height of 9 feet? *Justify* your answer algebraically.

18. A box is a rectangular prism with a volume of 768 cubic inches. The length of the box is 4 inches more than its height. Its width is the difference of 16 and its height.

a. Draw a diagram of the box and label its dimensions in terms of its height.

b. Write a polynomial that represents the volume of the box.

c. Use the polynomial from part (b) to find two sets of possible dimensions of the box.

d. Which set of dimensions results in a box with the least possible surface area? *Explain* your reasoning.